PUPC

Princeton University Physics Competition

November 19, 2017

Directions: The test consists of four problems, which you will be given 1.5 hours to complete. No collaboration is allowed.

Some useful test-taking hints:

- You may not be able to complete every problem. Keep moving; do what you know first.
- Make your answer clear by circling it.
- Use symbols rather than numbers wherever possible and check units.
- Wherever possible, check whether an answer or intermediate result makes sense before moving on.
- If you get stuck on an early part of a problem, check whether you can still do the later parts; some may be independent and doable.
- If you get stuck on an early part of a problem, and a later part depends on it, clearly define a symbol for the unknown answer and use it in later parts. However, keep in mind that we often give multiple parts to guide you through a problem.

To get full credit you need to show your work! Partial credit will also be awarded at the judges' discretion. **Good luck!**

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1 Warm-Up (15 points)

Each of the following problems is worth 5 points. Be sure to justify your reasoning for full credit!

a. **Pipe Betting** You release a hollow pipe of radius a and mass m, a solid sphere of radius $2a$ and mass m, and a solid pipe of radius a and mass $2m$ from rest at the same time from the top of an incline. Recall that the moment of inertia of a hollow cylinder is MR^2; the moment of inertia of a solid sphere is $\frac{2}{5}MR^2$; and the moment of inertia of a solid cylinder is $\frac{1}{2}MR^2$. Which arrives at the bottom first?

b. **Gravity × 2017** Consider a ring of 2018 evenly spaced point masses. The gravitational field at the center of the ring is 0; that is, a mass placed at point P in the center of the ring will experience no net force. Now suppose you remove one of the masses. What is the magnitude and direction of the resultant gravitational field at point P? Fig. 1 shows the scenario with 8 point masses.

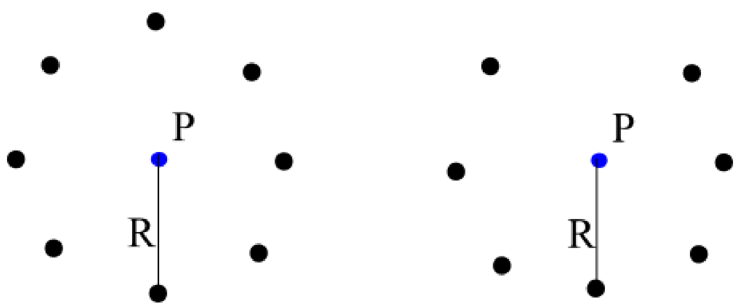
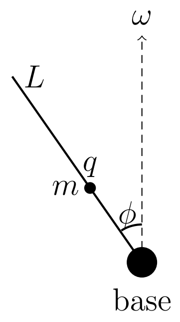


Figure 1: Ring of Masses, Before And After

c. **Stable or Unstable?** Consider the potential energy given by $U(x) = \sin xe^x$ in the interval $[0, 2\pi)$. What are the points of equilibrium? Identify each as stable or unstable.

2 Bead on Rotating Rod (25 points)

A bead of mass m is free to slide along a thin rod of length L tilted at angle ϕ to the vertical. The rod has a base point fixed to the ground and is spinning at constant angular velocity ω about the vertical (the dashed line). Gravity acts in the downwards vertical direction.



a. *(5 points)* If the rod rotates faster than a certain ω_c the bead will start to fly off. What is ω_c? Suppose that the bead is at length $q_0 > 0$ along the rod from the base point. Express your answer in terms of g, q_0, ϕ, m.

b. *(3 points)* Let $q(t)$ be the bead's distance along the rod from the base point as a function of time. For $\omega > \omega_c$ derive a differential equation for $q(t)$.

c. *(5 points)* Solve the equation you found in part b) for $q(t)$. You may assume the bead starts from rest at initial position q_0. (Hint: try a solution of the form $q(t) = A_1 e^{\Omega t} + A_2 e^{-\Omega t} + B$.)

d. *(1 point)* How long does it take for the bead to fly off the rod? You may leave your answer in terms of an equation involving t_f, the time at which it flies off.

e. *(4 points)* Once the bead flies off, find its **velocity** (with respect to the base point). In your answer, you may use the variables t_f, A, B, Ω defined above. Be clear about which components or directions of velocity you are identifying.

f. *(4 points)* From the moment the bead flies off, find the time T it takes to hit the ground. To simplify notation, define v as the velocity parallel to the rod at which the bead flies off. Express your final answer in terms of g, v, L, ϕ, m (you should not need to use A, B, Ω, t_f).

g. *(3 points)* How far from the center base point of the spinning rod will the bead land? Express your answer in terms of T, v, L, ϕ, m.

3 Disk Oscillations (25 points)

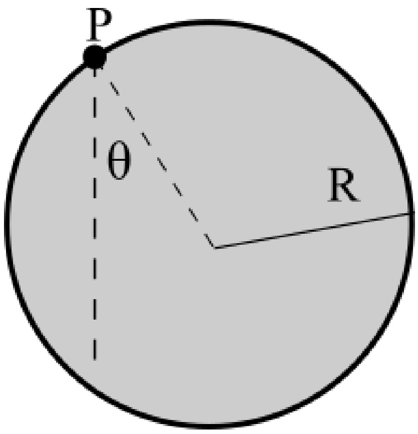
Consider a disk of negligible thickness. The mass density of the disk is $\rho(r) = kr$, where r is the distance from the center of the disk. Let the disk have radius R and mass M.

Part 1

a. *(3 points)* Find the mass M in terms of the other constants in the problem.

b. *(5 points)* Show that the moment of inertia of the disk about the axis through its center is $\frac{3}{5}MR^2$. To receive full credit, please show all of your work.

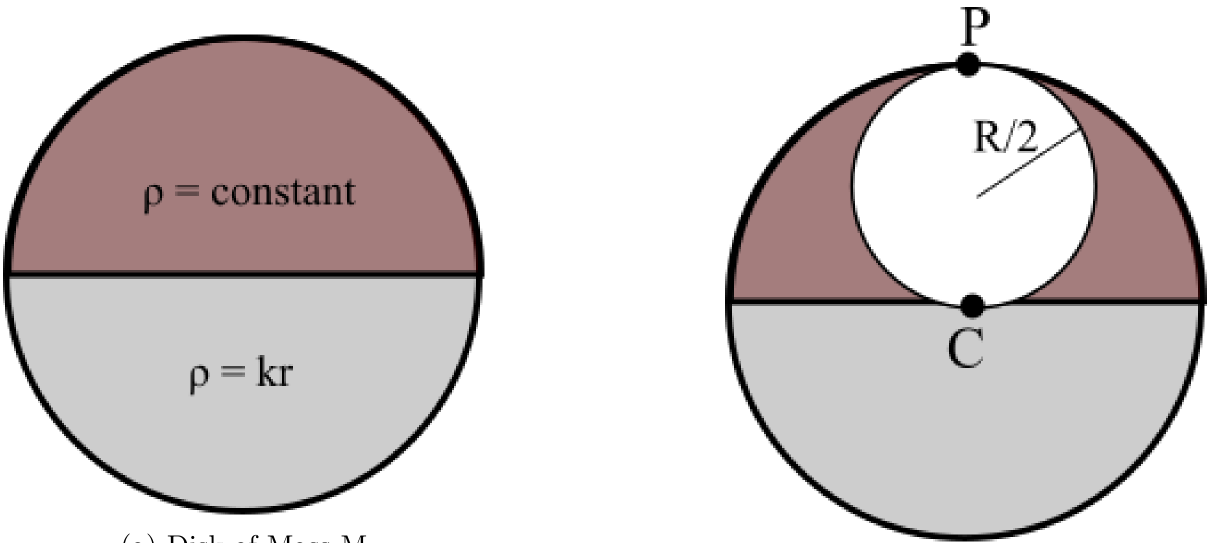
c. *(5 points)* Now consider the following setup:



The disk is pinned to a wall through the point P on the edge of the disk. If it is released from rest at an angle θ, where θ is very small, what is the frequency of the oscillations?

Part 2

Now consider another disk, also with mass M and radius R. The upper half of the disk has constant mass density ρ_1 and mass $M/2$, while the bottom half has linear mass density $\rho_2 = kr$ and mass $M/2$. (See Fig. 2a below.) The disk is pinned to a wall through point P; then, a circular hole of radius $R/2$ is cut from the disk that passes through both point P and the center C of the disk. (See Fig 2b.)



(a) Disk of Mass M

(b) Disk with Hole

Figure 2: Oscillating Disk in Part 2

a. *(6 points)* Calculate the moment of inertia of the new disk with hole with respect to C.

b. *(6 points)* Calculate the frequency about point P for the disk with hole.

4 Soap Bubble (20 points)

Air is blown vertically downwards to inflate a soap bubble. The bubble is suspended from a "frame" of radius r and negligible thickness. Assume the soap bubble remains a spherical shape except where it intersects the frame with the circular hole into which the air is blown. (See Fig. 3 below.)

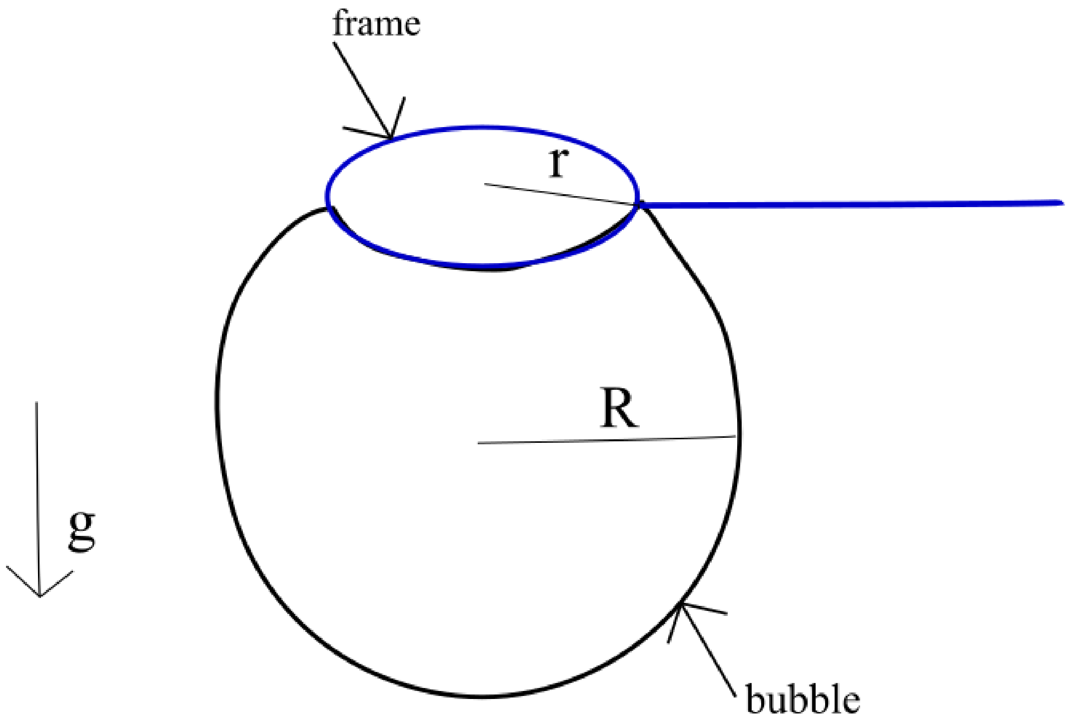


Figure 3: Soap Bubble

Given:

Constant air flow velocity: v

Constant air density: ρ

Constant mass per unit area of soap film: σ

Energy required to form a soap bubble of area A due to surface tension: γA

Surface area of a sphere with "cap" removed: $4\pi R^2 - \pi(r^2 + h^2)$

Volume of a sphere with "cap" removed: $\frac{4}{3}\pi R^3 - \frac{1}{3}\pi h^2(3R - h)$

where $h = R - \sqrt{R^2 - r^2}$

Find the equation of the final radius of the bubble, R_f, just after it detaches from the frame. You may ignore the movement of the bubble's center of mass during the process.

Hints: What is the force of surface tension? of air flow? of gravity? What is the relationship between these forces at the moment of detachment? Partial credit will be awarded for finding each force.