Introduction to Propositional Calculus

- Proposition: a statement that is either true or false

Propositional Calculus

1. Basic Notation
    1) P (antecedent) → Q (consequent); if P, then Q
    2) ~P (negation); not P
2. Structure

Assumptions (if assumptions are true, then the given proposition/argument must be true for the derivation to be sound)	Proposition number	Proposition expression (argument)	Rule of derivation used (justification)

To be Proved Notation: (premises) ⊦ (conclusion) e.g. P → Q, P ⊦ Q

3. Rules of Derivation (10)
    1) A: Rule of Assumption
       Used to introduce new propositions that depends on themselves to be true
       ex.) 1 (1) P   A
    2) MPP: Modus (ponendo) ponens
       ex.) 1 (1) P → Q   A
       2 (2) P   A
       1, 2 (3) Q   1,2 MPP □
    3) MTT: Modus (tolendo) tollens
       ex.) 1 (1) P → Q   A
       2 (2) ~Q   A
       1,2 (3) ~P   1,2 MTT □
    4) DN (Double negation): ~~P → P
       ex.) 1 (1) ~~P   A
       1 (2) P   1 DN □
    5) CP (Conditional proof): used to introduce hypotheticals e.g. P → Q
       ex.) 1 (1) P → Q   A
       2 (2) ~Q   A
       1,2 (3) ~P   1,2 MTT

(get rid of proposition no. 2 from the assumption because it becomes part of the propositon)

1 (4) ~Q→ ~P 2,3 CP □

6) &I (and-introduction)

ex.) 1 (1) P A

2 (2) Q A

1,2 (3) P & Q 1,2 &I □

7) &E (and-elimination)

ex.) 1 (1) P & Q A

1 (2) P 1 &E □

8) vI ('v' stands for 'vel,' the latin word for an inclusive 'or') (or-introduction)

- 'or' statements such as P v Q are true if: P is true, Q is true, both P and Q are true
- Propositions such as 'P v ~P' are always true. By definition, propositions such as 'P' are either true or false, thus either 'P' or '~P' must always be true

9) vE (or-elimination)

ex.) 1 (1) P v Q A

2 (2) P A

3 (3) Q A

2 (4) Q v P 2 vI

3 (5) Q v P 3 vI

1 (6) Q v P 1,2,3,4,5 vE □

10) RAA (reducio ad absurdum)

Use the premise to make a contradiction such as P & ~P (a paradox), then reject premise

P → Q, P → ~Q ├ ~P

1 (1) P → Q A

2 (2) P → ~Q A

3 (3) P A

1,3 (4) Q 1, 3 MPP

2,3 (5) ~Q 2, 3 MPP

1,2,3 (6) Q & ~Q 4, 5 &I

1,2 (7) ~P 3, 6 RAA □