2015 WMI Student Seminars

"Pythagorean Theorems and Triples"

Donghun Cho(donghuncho830@gmail.com)

August 2015

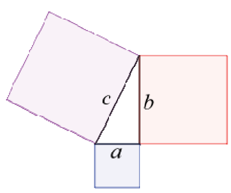
1. Introduction

Hello, everyone! The seminar, or just brief speech that I will give to you is about very well-known geometric theorem, Pythagorean Theorem. This very basic theorem, as you do know already, is very useful tool for solving many geometric problems.Today, with this interesting theorem, I would like to present to you some basic knowledge and proofs of this theorem, Pythagorean triples, and how this theorem can be used in all kinds of area. Also, more importantly, I want to show you that you can get so much intriguing facts out of very simple and basic theorem. Surprising power of math.

2. Proofs of Pythagorean Theorem

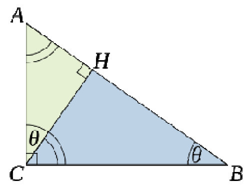
Before I show different types of proofs, I would explain what the Pythagorean Theorem is, although you would probably all know this already. Pythagorean Theorem states that the square of the hypotenuse (the side opposite to the right angle) is equal to the sum of the squares of the other two sides.

$$a^2 + b^2 = c^2$$



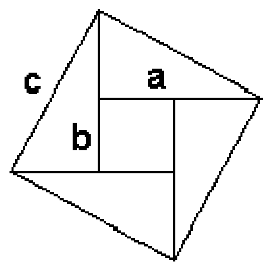
Now, I will show you few different proofs of this fundamental thorem.

i. Geometric Proof



From the figure above, you can see that ABC, ACH, and CBH are all similar triangles. So $\frac{BC}{AB} = \frac{BH}{BC}$, $\frac{AC}{AB} = \frac{AH}{AC}$. $BC^2 = AB \times BH$ and $AC^2 = AB \times AH$. By adding these two equations, we get $BC^2 + AC^2 = AB \times (BH + AH) = AB^2$. Finally, we can conclude $AB^2 = AC^2 + BC^2$.

ii. Algebraic Proof



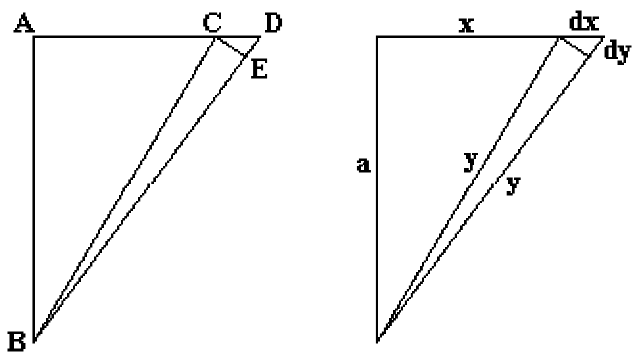
$$(\text{Area of big square}) = (\text{Area of small square}) + 4 \times (\text{Area of triangle})$$

$$c^2 = (a - b)^2 + 4 \times \frac{ab}{2}$$

$$c^2 = a^2 + b^2$$

iii. Differential Proof

The two proofs I have presented above are very commonly used proofs, but the proof I will show right now is not introduced very often. This proof is using differential equation.



ABC and ECD are similar triangles.

$$\frac{x}{y} = \frac{dy}{dx}$$

$$\mathrm{y} \times \mathrm{dy} - x \times dx = 0$$

By solving differential equation(doing integration on both part), we get

$$y^2 - x^2 = C$$

The constant can be determined from the initial condition for x=0.

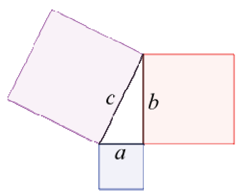
$$\text{Since } \mathrm{y}(0) = \mathrm{a}, \ \mathrm{C} = a^2$$

$$\text{So, } y^2 = x^2 + a^2$$

Now, I have displayed only three proofs of the Pythagorean Theorem. All of the proof are approached from different ways, geometric, algebraic, and differential. However, it is surprising that there are still 40 more proofs that have been discovered. It is fascinating that there could be 43 proofs of one little thorem.

3. Pythagorean Triples

In this section, I would like to talk about Pythagorean Triples. A Pythagorean triple consists of three positive integers a,b, and c, such that $a^2 + b^2 = c^2$. Such a triple is commonly written (a,b,c), and some of the examples are (3,4,5), (5,12,13), (8,15,17), and so on. I would like to show that these numbers are not just random, they have certain patterns.



All triples can be written as (a,b,c) where $a^2 + b^2 = c^2$, a,b, and c are coprime, and b and c have opposite parities(one is even and one is odd). If b and c are both even, then a,b, and c are not coprime. If b and c are both odd, the equation would equate an even to an odd. So b and c must have opposite parities. Since $a^2 + b^2 = c^2$, we can get $c^2 - a^2 = b^2$ and so $(\mathrm{c} - \mathrm{a})(\mathrm{c} + \mathrm{a}) = b^2$. So, $\frac{a+c}{b} = \frac{b}{c-a}$. Since $\frac{a+c}{b}$ is rational, we can set it to equal $\frac{m}{n}$ in lowest terms, then $\frac{b}{c-a} = \frac{m}{n}$. So $\frac{c-a}{b} = \frac{n}{m}$. $\frac{c}{b} + \frac{a}{b} = \frac{m}{n}$ and $\frac{c}{b} - \frac{a}{b} = \frac{n}{m}$.

By adding and subracting two equations above, we can get $\frac{c}{b} = \frac{m^2+n^2}{2mn}$ and $\frac{a}{b} = \frac{m^2-n^2}{2mn}$. Finally, we can see that $\mathrm{a} = m^2 - n^2$, $\mathrm{b} = 2mn$, and $\mathrm{c} = m^2 + n^2$ satisfy the Pythagorean Theorem. If you first look at the lists of the Pythagorean triples, you might think that you actually have to do the trial and error for numerous numbers and check if it works or not. However, there are much simpler way than calculating all of the numbers.

4. Conclusion

In this very brief presentation, I have dealt with quite fundamental theorem, Pythagorean Theorem. I wanted to show you that even with this commonly used theorem, if you think just a little bit deeper, you can learn more interesting things. Although it wasn't very professional presentation, I hope you gained little more interest in mathematics than you had before. Thank you.

7 days after the eggs were washed free of acid and placed in culture at 28C. As performed, the propidium iodide assay was not successful in identifying the majority of dead eggs when compared to the larval development assay. The larvation assay did show that the treatment of the eggs with the acids markedly reduce viability with 5 minutes of contact, and with 60 minutes contact the combinations acids all reduced egg viability by 97%. The work suggests that optimization of the application methods, contact time, temperature, and surfactant may allow this to become a viable means of *A. galli* control in organic facilities.

2. Introduction

The nematode *Ascaridia Galli* is a large worm, whose infection causes weight depression in the host (W. Malcolm Reid et al. 1958). One of the most striking effects of infection by *Ascaridia Galli* is the occasional finding of this parasite inside chicken's egg. Numerous reports of this phenomenon have been made in the literature (W. M., J. L. Mabon 1973). It is suggested that the worms migrate up the oviduct via the cloaca, with subsequent inclusion in the egg (Calnek, B. W 1997). Organic facilities can't use Anthelmintic to control these parasites. It is shown that these acids when they come into contact with the eggs of the swine ascarid, *Ascaris suum*, have deleterious effects when the acids are at a pH below the pK_a of the given acid(Butkus, M. A et al 2011). Thus, usage of short chain fatty acids in controlling the viability of *Ascaridia Galli* can be helpful in organic facilities since short chain fatty acids are approved for use in organic facilities. Two different hypotheses are constructed in this work: Short chain fatty acids will reduce the viablility of *Ascaridia* eggs and propidium iodide dye permeability will identify viable *Ascaridia* eggs without the need to wait seven days until eggs larvate.

3. Materials and Methods

Several tests involving various treatments were administrated using 950µl of the purposed treatment as well as 50µl of the *Ascaridia* egg sample which has concentration of 23.2 eggs/µl, purified from chicken feces. Thus, 1160 eggs are used in each samples.

Combinations of 1.5 M butanoic, pentanoic, or hexanoic acids in 18 mM Tween 20 were used in seven experiments: 1.5M butanoic, 1.5M pentanoic, 1.5M hexanoic, 1.5M butanoic and pentanoic, 1.5M butanoic and hexanoic, 1.5M pentanoic and hexanoic, and 1.5M butanoic, pentanoic, and hexanoic.

After eggs were added to acid solution for a final volume of 1 ml in 1.2ml microfuge tube, they were vortexed for three seconds and placed in a heating block at 37℃ with agitation. At various sampling times, the tubes were removed and centrifuged at centrifugal force1200g for a minute to pellet the eggs. The acid was suctioned off without disturbing the egg pellet, and the eggs were washed six times with phosphate buffer (pH 7.0, 10mM). Unless otherwise stated all experiments were carried out in two replications for each exposure time, temperature, acid concentration and acid with surfactant.

Method 1 Propidium Iodide Permeable Dye Assay

Half of the eggs in microfuge tube were stained with Propidium Iodine. Propidium Iodide is fluorescent dye, binding intercalatively to both DNA and RNA with little base-pair specificity. Pink Propidium Iodide Fluorescence under UV excitation is indicative of cell permeabilization. Both DNA and RNA are stained, giving a red-pink luminescence under excitation form a triple-filter set. Because this dye permeates outer shell and nuclear membrane to get through DNA, we can notice the viability of *Ascaridia* eggs by looking inside fluorescence microscope. Figure 1 shows the difference in color of viable *Ascaridia* eggs and non-viable *Ascaridia* eggs in fluorescence.

Method 2 Larvation

After washing, a half of all eggs were transferred to 12 well culture plates with the addition of H_2SO_4 solution to retard mold growth during incubation. The plate, wrapped in a wet paper towel in a plastic box, was statically incubated at 28℃ (82.4℉) for 7 days. The eggs were counted and scored as larvated (viable) or nonlarvated (not viable). Figure 2 demonstrates the daily egg development of *Ascaridia* egg from day 0 to day 19 showing the larvated form of *Ascaridia* egg. Figure 3 shows the difference in existence of larvae in viable *Ascaridia* eggs and non-viable *Ascaridia* eggs.

4. Results

Method 1 Propidium Iodide Permeable Dye Assay

As shown in Figure 4, the viability of *Ascaridia* egg decreases as exposure time pasts by. Compared to control, butanoic, pentanoic, and hexanoic acids all reduced the viability of the *Ascaridia* eggs to below detectable limits. However, there is certainly a viability difference between pentanoic acid and even-number acids, which are butanoic acid and hexanoic acid. While viability of *Ascaridia* eggs in pentanoic acid exposures at 60 minutes is about 88%, the viability of *Ascaridia* eggs in butanoic acid exposures is 81% and the viability of *Ascaridia* eggs in hexanoic acid exposure is 82%.

Continuing to the results of each acid exposure, Figure 5 which is the result of combination of each two acids exposure also shows the decrease in viability of *Ascaridia* eggs compared to control. Also shown in Figure 4, pentanoic acid has less effect on decreasing viability on *Ascaridia* eggs. The combination of different types of fatty acids actually decreases the viability of *Ascaridia* Eggs that viability of *Ascaridia* eggs in butanoic and pentanoic acid exposures at 60 minutes is 82% and viability of *Ascaridia* eggs in pentanoic and hexanoic acid exposures at 60 minutes is 79%. Without pentanoic acid, viability of *Ascaridia* eggs in butanoic and hexanoic acid exposures at 60 minutes is the least, 74%.

As shown in Figure 6, Combination of all three acids gave us similar result that viability of *Ascaridia* eggs in butanoic, pentanoic and hexanoic acid exposures at 60 minutes is 75% while the viability of *Ascaridia* eggs in control at 60 minutes is about 90%. Although we could gain data right after the treatments are done to *Ascaridia* eggs, the viability data by propidium iodide permeable dye assay was not significant enough.

Method 2 Larvation

Different from that of viability by propidium iodide permeable dye assay, Figure 7,8,9, graphs of viability by larvation, have the axis dealing 0% to 100%. That is, the result shows more decreasing effects of fatty acids in viability of *Ascaridia* eggs. Figure 7 suggests that viability of *Ascaridia* eggs exposed on pentanoic acid is the highest for 24% at 60 minutes exposure. It means the lowest decrease on viability since the viability of *Ascaridia* eggs exposed on butanoic acid is 5.37% at 60 minute exposure and viability of *Ascaridia* eggs exposed on hexanoic acid is 5.62% at 60 minutes exposure.

As shown in Figure 8, results of combination of each two acids exposure also reveals high decrease in viability of *Ascaridia* eggs. In this result, the viability of *Ascaridia* eggs exposed in butanoic and hexanoic acid is the lowest which records 4.4% at 60 minute exposure. The viability of *Ascaridia* eggs in butanoic and pentanoic acid exposures is 6% and the viability of *Ascaridia* eggs in pentanoic and hexanoic acid exposure is 8%. All three results definitely show that the fatty acids actually have the effect on *Ascaridia* eggs' viability.

Figure 9 also clearly demonstrates the difference of *Ascaridia* egg viability between the control and butanoic, pentanoic, and hexanoic acid combination exposure. While 90% of eggs in the control survived after 60 minutes exposure, only 3% of eggs in butanoic, pentanoic and hexanoic exposure survived after 60 minute exposure at 37 °C in shaking condition.

5. Discussion

In conclusion, fatty acids exposure has decreased the viability of the *Ascaridia* eggs. The propidium iodide assay was not successful in identifying the majority of dead eggs when compared to the larval development assay. Even number acids, butanoic and hexanoic acid, are more effective than pentanoic acid in decreasing the viability of *Ascaridia* Eggs. These findings are consistent with those of Paggi and Fay (1996), where acetic acid was less effective than propanoic and butanoic acids against *Streptococcusbovis*. For the future studies, we should further test the fatty acid disinfection effects on *Ascaridia* eggs in manure matrix and contaminated egg producing equipment. According to the result, it is possible to develop an organic pesticide that can be sprayed in the cages of the chickens using fatty acids.

6. References

Reid, W. Malcolm, and James L. Carmon. "Effects of Numbers of *Ascaridia Galli* in Depressing Weight Gains in Chicks." *The Journal of Parasitology* 44.2 (1958): 183. Web.

Reid, W. M., J. L. Mabon, and W. C. Harshbarger. "Detection of Worm Parasites in Chicken Eggs by Candling." *Poultry Science* 52.6 (1973): 2316-324. Web.

Calnek, B. W. *Diseases of Poultry*. Ames, IA, USA: Iowa State UP, 1997. Print.

Butkus, M. A., K. T. Hughes, D. D. Bowman, J. L. Liotta, M. B. Jenkins, and M. P. Labare. "Inactivation of Ascaris Suum by Short-Chain Fatty Acids." *Applied and Environmental Microbiology* 77.1 (2010): 363-66. Web.

Paggi, R. A., and J. P. Fay. 1996. Effect of short-chain fatty acids on growth of the ruminal bacterium *Streptococcus bovis*. J. Gen. Appl. Microbiol. 42:393–400.

7. Figures Legends

Figure 1. Viable *Ascaridia* eggs (left) and non-viable *Ascaridia* eggs (right) stained in propidium iodide

Figure 2. Daily egg development of *Ascaridia* egg from day 0 to day 19

Figure 3. Larvated *Ascaridia* eggs (left) and Non-larvated *Ascaridia* eggs (right) in

Figure 4. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic, 1.5M pentanoic, 1.5M hexanoic, each in 18 mM Tween 20, by propidium iodide permeable dye assay

Figure 5. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic and pentanoic, 1.5M butanoic and hexanoic, 1.5M pentanoic and hexanoic, each in 18 mM Tween 20 by propidium iodide permeable dye assay

Figure 6. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic, pentanoic, and hexanoic in 18 mM Tween 20, by propidium iodide permeable dye assay

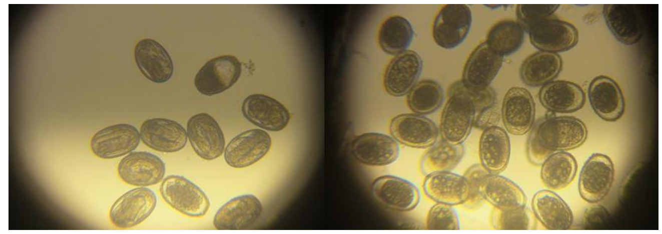
Figure 7. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic, 1.5M pentanoic, 1.5M hexanoic, each in 18 mM Tween 20, by larval development assay

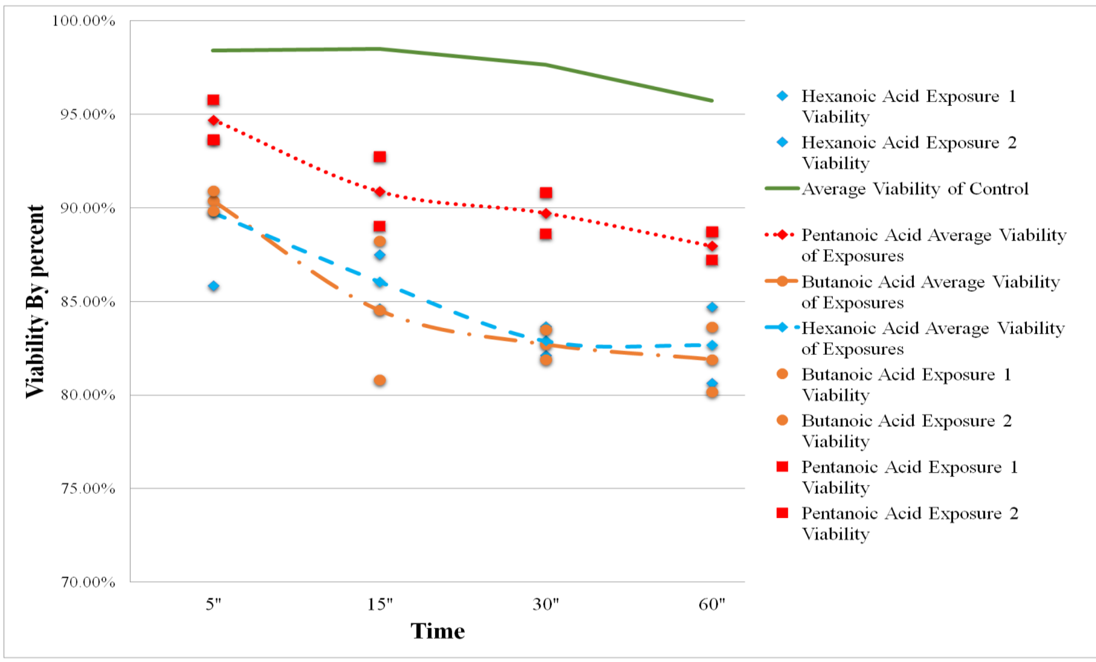
Figure 8. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic and pentanoic, 1.5M butanoic and hexanoic, 1.5M pentanoic and hexanoic, each in 18 mM Tween 20 by larval development assay

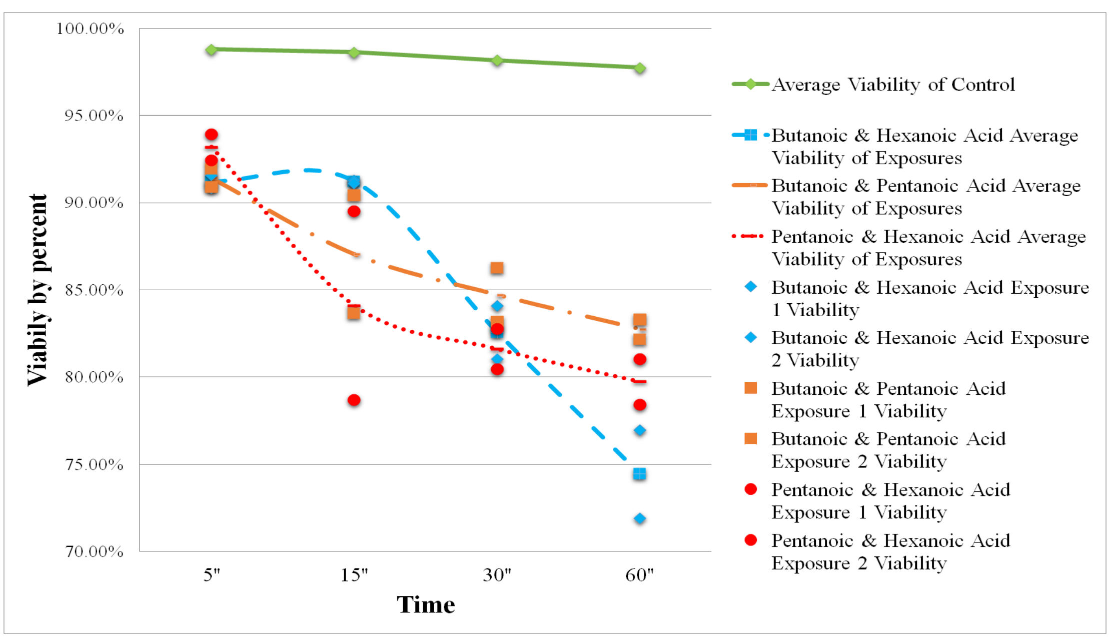
Figure 9. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic, pentanoic, and hexanoic in 18 mM Tween 20, by larval development assay

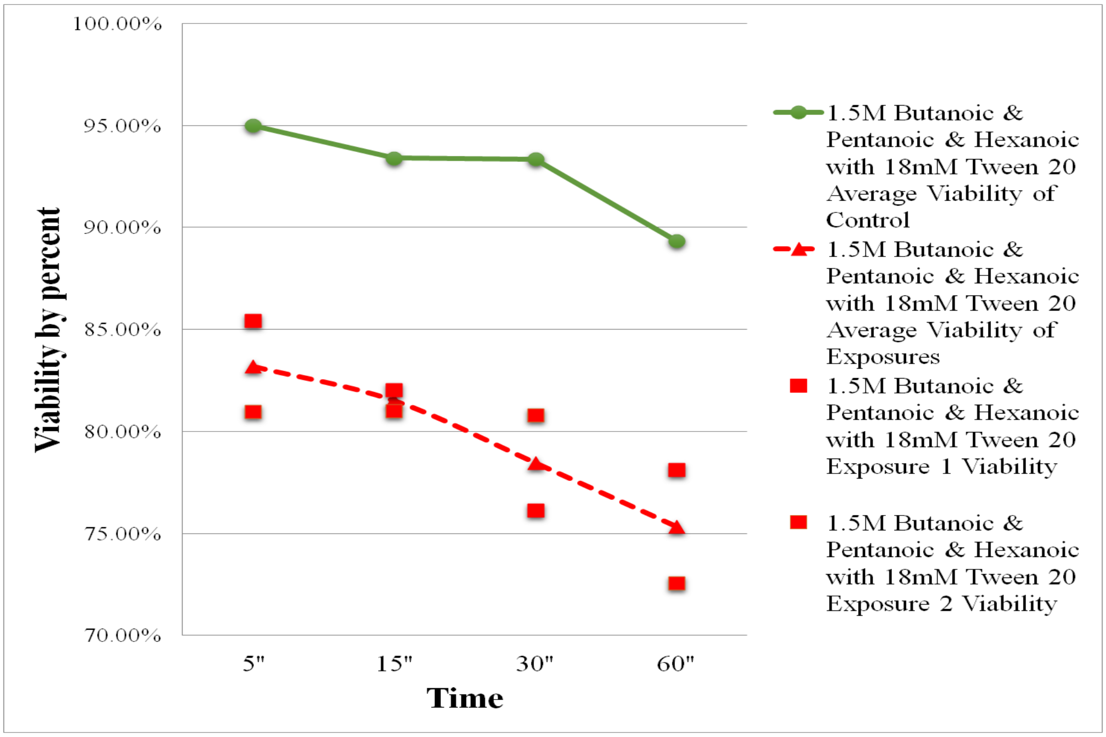
8. Figures

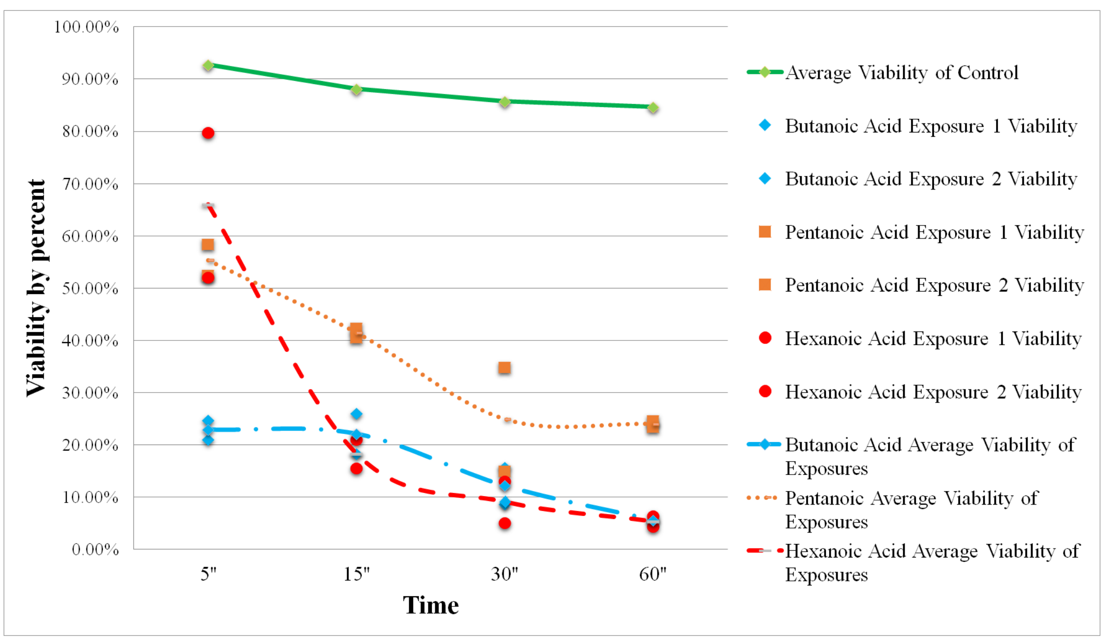

0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19

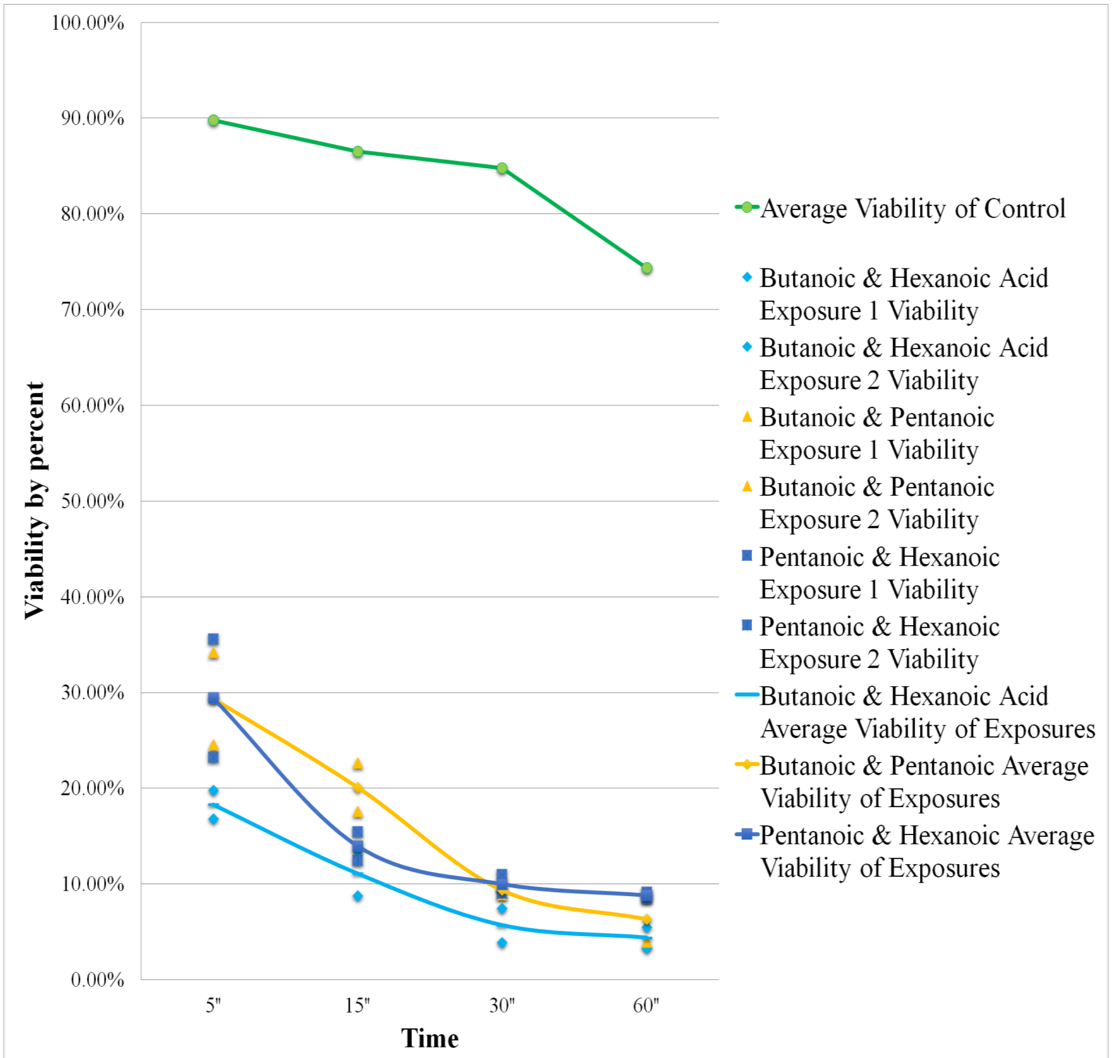


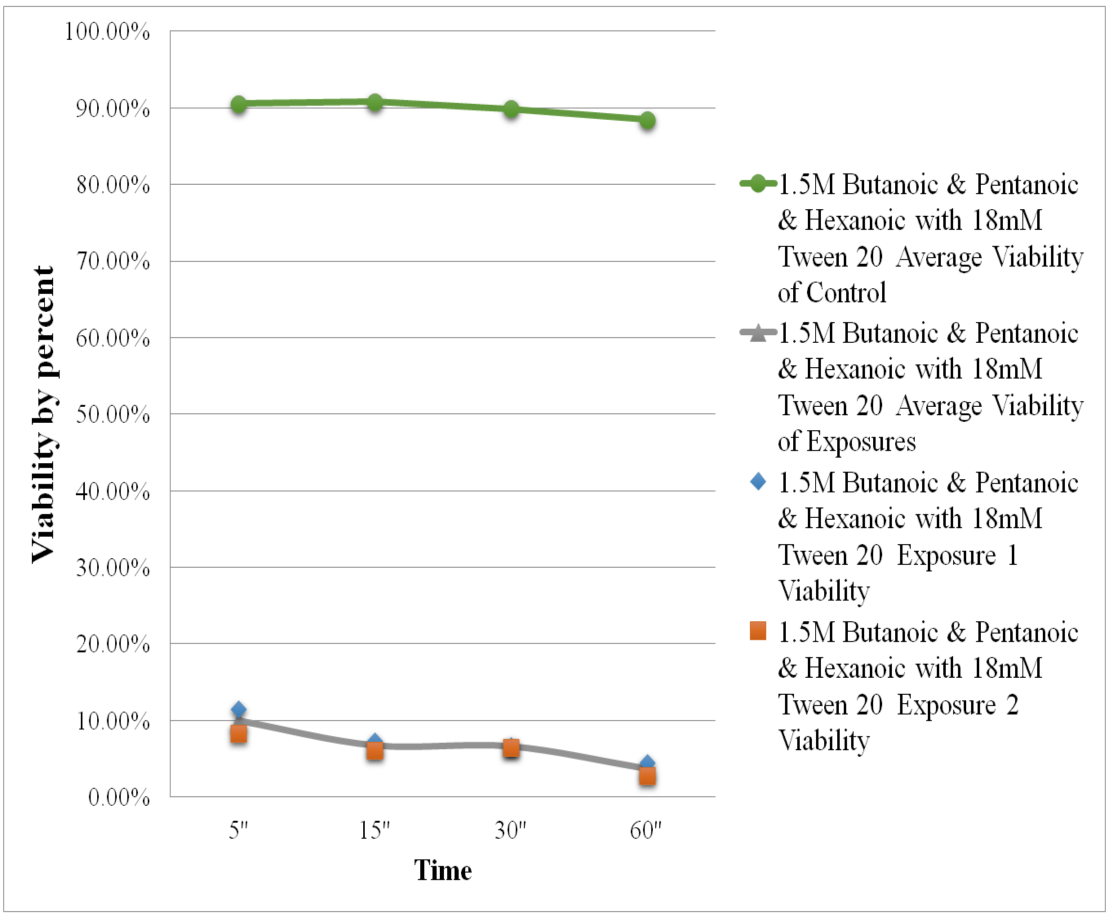

100.00%
95.00%
90.00%
85.00%
80.00%
75.00%
70.00%
Viability By percent
5"
15"
30"
60"
Time
Hexanoic Acid Exposure 1 Viability
Hexanoic Acid Exposure 2 Viability
Average Viability of Control
Pentanoic Acid Average Viability of Exposures
Butanoic Acid Average Viability of Exposures
Hexanoic Acid Average Viability of Exposures
Butanoic Acid Exposure 1 Viability
Butanoic Acid Exposure 2 Viability
Pentanoic Acid Exposure 1 Viability
Pentanoic Acid Exposure 2 Viability


100.00%
95.00%
90.00%
85.00%
80.00%
75.00%
70.00%
Viabily by percent
5"
15"
30"
60"
Time
Average Viability of Control
Butanoic & Hexanoic Acid Average Viability of Exposures
Butanoic & Pentanoic Acid Average Viability of Exposures
Pentanoic & Hexanoic Acid Average Viability of Exposures
Butanoic & Hexanoic Acid Exposure 1 Viability
Butanoic & Hexanoic Acid Exposure 2 Viability
Butanoic & Pentanoic Acid Exposure 1 Viability
Butanoic & Pentanoic Acid Exposure 2 Viability
Pentanoic & Hexanoic Acid Exposure 1 Viability
Pentanoic & Hexanoic Acid Exposure 2 Viability


100.00%
95.00%
90.00%
85.00%
80.00%
75.00%
70.00%
Viability by percent
5"
15"
30"
60"
Time
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Average Viability of Control
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Average Viability of Exposures
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Exposure 1 Viability
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Exposure 2 Viability


100.00%
90.00%
80.00%
70.00%
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
Viability by percent
5"
15"
30"
60"
Time
Average Viability of Control
Butanoic Acid Exposure 1 Viability
Butanoic Acid Exposure 2 Viability
Pentanoic Acid Exposure 1 Viability
Pentanoic Acid Exposure 2 Viability
Hexanoic Acid Exposure 1 Viability
Hexanoic Acid Exposure 2 Viability
Butanoic Acid Average Viability of Exposures
Pentanoic Average Viability of Exposures
Hexanoic Acid Average Viability of Exposures


100.00%
90.00%
80.00%
70.00%
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
Viability by percent
5"
15"
30"
60"
Time
Average Viability of Control
Butanoic & Hexanoic Acid Exposure 1 Viability
Butanoic & Hexanoic Acid Exposure 2 Viability
Butanoic & Pentanoic Exposure 1 Viability
Butanoic & Pentanoic Exposure 2 Viability
Pentanoic & Hexanoic Exposure 1 Viability
Pentanoic & Hexanoic Exposure 2 Viability
Butanoic & Hexanoic Acid Average Viability of Exposures
Butanoic & Pentanoic Average Viability of Exposures
Pentanoic & Hexanoic Average Viability of Exposures


100.00%
90.00%
80.00%
70.00%
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
Viability by percent
5"
15"
30"
60"
Time
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Average Viability of Control
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Average Viability of Exposures
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Exposure 1 Viability
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Exposure 2 Viability

2015 WMI Student Seminars

"Cycloids in Our Lives"

Hyunji Kim (didikim1999@gmail.com)
August 2015

1. Introduction

There are many wonders in our world, both from the technological innovation of mankind and the technicalities of the natural world that we simply take for granted. But upon closer look, you will be able to find many math concepts that are hidden behind. Today, I would like to focus on one of these hidden concepts, which is very pervasive in our lives but only a few know about. Well, without further a due, let us begin!

2. What is a Cycloid?

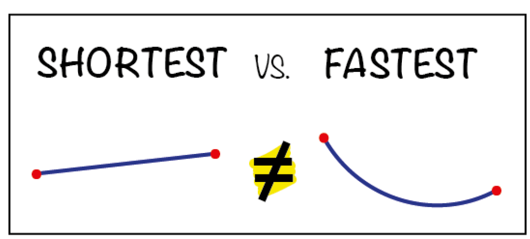
First, let us start off with a simple question.
"What is the shortest distance from one point to another?"

No one will have trouble coming up with the answer. It's a straight line.

Let me ask you a slightly different question.
"What is the *fastest* way from one point to another?"

Now you may be thinking *'That's also a straight line!'*

But in fact, it is not. Think back at some of the slides you have ridden. Were they all straight? You may recall that the fun, or the more thrilling ones were those that were curved. This is because they were faster. As you may have guessed, the fastest way between two points is reached through a *curve.* Not just any random curve, but a special curve called a 'Cycloid'.



So what exactly, is a cycloid?
By definition, it is a curve generated by a point on a circumference of a circle that rolls without slipping, on a straight line.

2015 WMI Student Seminars

"Generating Functions"

Phillip Yoon (phillip_yoon@hotmail.com)
August 2015

1. Introduction

Welcome! It's a pleasure to be able to present my topic on generating functions. My goal is that you will leave knowing what generating functions are and some classic examples of how they can be used. For example, we can find that the nth term of the Fibonacci sequence is given by $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. It's a surprising formula, yet you will leave understanding how that formula was derived! Generating functions give us a useful and aesthetically pleasing way to solve many problems that may otherwise be quite difficult.

2. Recursive Formulas

Before we jump right into the exact formula for the Fibonacci sequence, we need some background first. The Fibonacci Sequence is well known: $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ The rule is that the next term is the sum of the two previous terms. How would we write the formula for this sequence? We can use a recursion, meaning that we can write the sequence in terms of itself. We have $a_n = a_{n-1} + a_{n-2}$, meaning that any term is the sum of the two terms before it. For example, $a_0 = 0$, $a_1 = 1$, and $a_2 = 1$, and $a_2 = a_1 + a_0$.

Recursive formulas give us many more options to describe sequences. For example, for the sequence $1, 7, 37, 187, 937, \ldots$, the formula $a_n = 5a_{n-1} + 2$, where $a_0 = 1$, looks a lot nicer than $a_n = (\frac{3}{2} \cdot 5^n - \frac{1}{2})$. However, it is very often useful to be able to move from a recursive definition to an exact one, one where all we need is the value of n to calculate a_n. We will demonstrate this with the Fibonacci sequence after we learn about geometric series.

2015 WMI Student Seminars

"A Dilemma and Paradox that can affect your life"

Sungwon Hong (icebluehong@naver.com)

August 2015

1. Introduction

Every day, we face conflicts. Sometimes we face big problems, such as deciding which colleges to apply, whereas at other times, we face smaller conflicts, such as deciding whether to drink coke or cider. The following dilemma and paradox may not provide a direct solution to your agonies; however they may help you to sagaciously plan your life.

Scientists have discovered dice made of sheep bones, which are thought to have been used by people in B.C. 3500. Scientists conjecture these bones are the origin of "probabilities." In the Middle Ages, people believed studying probabilities were an action against God. So, probabilities research did not take off until the 17th century, starting with calculating the winning rates in gambling.

Under this context, I will present the most recent findings in probability research pertaining to Simpson's Paradox and the Monte Hall Dilemma.

2. Paradox and Dilemma in Probability

A. Simpson's paradox

To understand easily, let's have an example.

2015 WMI Student Seminars

"Cryptography and Math"

Yongchan Hong (yongchana@gmail.com)
August 2015

1. Introduction

Hello ladies and gentlemen! Welcome to the world of cryptography! Cryptography is an essential field of study that is utilized in everyday life. Even though it is not shown directly, cryptography is used in WIFI, Bitcoin, SSL, digital signatures and etc. You cannot imagine a world without using cryptography.

Then what is exactly "Cryptography"? There are various definitions for cryptography, but I chose the most clear and concise one. Cryptography is a method of storing and transmitting data in particular form so only those for whom it is intended can read and process it. This cryptography had started long from ancient times, Rosetta stone used in order to keep its information secret. In order to keep each person's information safe, cryptography uses mathematics in various ways. In this lecture, we will cover mathematics applied in basic cryptography, such as Caesar, Affine Caesar, Hill and RSA.

2. Classification of Cryptography

Cryptography is a subject that has various fields inward. Cryptography is mainly divided into two, which is transposition and substitution. Transposition is changing the word order of the message. Substitution is changing the word itself. This substitution is divided into two, which is by code and cipher. Code is changing the word itself, while cipher is changing single characters. In this lecture, we will only discuss about coding in cipher.

<table>
<tr><td rowspan="3">Cryptography</td><td>Transposition</td><td></td></tr>
<tr><td rowspan="2">Substitution</td><td>Code</td></tr>
<tr><td>Cipher</td></tr>
</table>

3. Basics for Cryptography

Before we learn how math is used in cryptography, we should learn about basic function of cryptography. Cryptography uses P, C, E and D to express its function. P is plain text. Plain text is the text that we wish to code. C is cipher text. Cipher text is the encoded plain text. E is encryption function, which is the function that we use in order to change plain text to cipher text. D is decryption text, which is the function that we use in order to change cipher text to plain text.

We show this function as:

E(P)=C

D(C)=P

To give example, let us say plain text is “APPLE.” If we move each alphabet to next alphabet, we can encode this into “BQQMF.” This text is cipher text. So,

E(APPLE)=BQQMF

D(BQQMF)=APPLE

As we now understand the basics of cryptography, let us dive into mathematics that resides inside the cryptography.

4. Caesar Cipher

The Caesar cipher is one of the early known and simplest encoding. It was used by Julius Caesar in protecting military messages. By using Caesar Cipher, Caesar could be successful in Rome. Caesar cipher is shifting the number by certain amount “t”. It is usually used in alphabet, and it is encoded by:

A->0, B->1, C->2, D->3…Z->25

And shift each number into certain amount and get new alphabet. The case “APPLE” I used to explain in “Basics for Cryptography” is also using Caesar Cipher. To explain in function, E(P)=(P+t, mod M)

D(C)=(C-t, mod M)

Let me explain what this means by using the “APPLE” example again. M in this place is 26, as we are using alphabetical order and changing those numbers into 0–25. If we say t=2, we should move our alphabets into 2 next alphabet.

E(P)=(P+2, mod 26)

As A=0, E(0)=(2, mod 26)=C

As P=16, E(16)=(18, mod 26)=R

As L=12, E(12)=(14, mod 26)=N

As E=5, E(5)=(7, mod 26)=G

E(APPLE)=CRRNG

So, we can get Cipher Text of CRRNG. In order to decipher this Cipher text CRRNG, we can simple subtract 2 (which is t) and get out plain text back. Caesar Cipher is most simple encoding that we can do.

5. Affine Caesar Cipher

This cipher is going one step further than Caesar Cipher. We multiply certain number “a” in plain text and add “t”. To show in function:

E(P)=(aP+t, mod M)

D(C)=((C-t)/a, mod M)

Caesar Cipher is the special case when a is 1. Let us use “APPLE” again. This time, we can encode by a=2, and t=1.

E(P)=(2P+1, mod 26)

As A=0, E(0)=(1, mod 26)=B

As P=16, E(16)=(33, mod 26)=H

As L=12, E(12)=(25, mod 26)=Z

As E=5, E(5)=(11, mod 26)=K

E(APPLE)=BHHZK

This Affine Caesar Cipher is hard to decode than Caesar Cipher. Now, we will look through other ciphers that are complex.

6. Hill Cipher

Hill Cipher is introduced in 1929 by Lester Hill, and this Hill cipher is a poly-alphabetic cipher that uses matrices to encode plaintext messages. This cipher is using certain n×n key matrix that is pre chosen.

In this cipher, we will use 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that is invertible mod 26. We should acknowledge that Det(A)=ad-bc (mod 26).

This time, as it should be multiple of 2, let us encode "LOVE" by using key matrix $\begin{bmatrix} 3 & 7 \\ 9 & 10 \end{bmatrix}$. If we change LOVE in number, it is 12, 15, 22, 5. If we perform Hill cipher:

$$\begin{bmatrix} 3 & 7 \\ 9 & 10 \end{bmatrix} \begin{bmatrix} 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 141 \\ 258 \end{bmatrix} \text{ mod } 26$$

$$\begin{bmatrix} 3 & 7 \\ 9 & 10 \end{bmatrix} \begin{bmatrix} 22 \\ 5 \end{bmatrix} = \begin{bmatrix} 101 \\ 248 \end{bmatrix} \text{ mod } 26$$

So, the numerical value of the cipher text is 11, 24, 23, 14

We can get cipher text KXWN

This hill cipher looks complex compare to other encoding methods! Then how can we decode Hill cipher? Decrypting the Hill Cipher is quite complex first, we should use A^{-1}matrix. A^{-1} is $\det(A)^{-1} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ mod m. We should then multiply cipher text in order to get plain text again. If we perform decrypting action:

$A^{-1}=19^{-1}\begin{bmatrix}10 & -7\\ -9 & 3\end{bmatrix}$ mod 26

19^{-1}=11, as 19*11=1 mod 26

$A^{-1}=\begin{bmatrix}110 & -77\\ -99 & 33\end{bmatrix}=\begin{bmatrix}6 & 1\\ 5 & 7\end{bmatrix}$ mod 26

If we put cipher text 11, 24, 23, 14

$\begin{bmatrix}6 & 1\\ 5 & 7\end{bmatrix}\begin{bmatrix}11\\ 24\end{bmatrix}=\begin{bmatrix}90\\ 223\end{bmatrix}$ mod 26

$\begin{bmatrix}6 & 1\\ 5 & 7\end{bmatrix}\begin{bmatrix}23\\ 14\end{bmatrix}=\begin{bmatrix}152\\ 213\end{bmatrix}$ mod 26

We can get plain text number 12, 15, 22, 5

We can get plain text LOVE.

This Hill cipher could be used in any word or phrase that is multiple of n. By cutting in n words, we can encode in n×n matrix. How is this ciphering method?? Now we will talk about the most complex ciphering method: RSA.

7. RSA Cipher

RSA cipher is the first encryption that made electric signal possible. This encryption was made by Ron Rivest, Adi Shamir and Leonard Adleman in 1977, and based on their name the "RSA" name has been made. Unlike other encrypting methods, RSA methods use two keys. This key means constant that can open and close the message. There is public key that is used to encrypt message and private key that is allowed to certain person to decrypt it. We should first understand about key generation.

We should choose p and q that is two distinct co-prime numbers.

Compute N=pq

Compute φ(N)=(p-1)(q-1). This φ is Euler's totient function. This value is private.

Search e that is smaller than φ(N) and co-prime to φ(N).

Search d that is de≡1 mod φ(N)

In here, (N, e) is public key and (N, d) is private key. It is important to erase p and q in order to prevent someone to guess d and e.

For encryption, we undergo this procedure:

$C=p^e \bmod N$

For decrypting, we undergo this procedure:

$P=c^e \bmod N$

This C is cipher text, and P is plain text. Do not forget this! This is quite confusing? So we will now discuss about actual performance for RSA.

Select two different prime numbers.

Select p=61, q=53

Compute N=pq

N=61×53=3233

Compute φ(N)=(p-1)(q-1).

φ(3233)= (61-1)(53-1)= 3120

Select any number e between 1<e<3120 and e that is prime to φ(N)

Select e=17

Compute d by the formula that is shown above.

17×2753= 46801= 1 (mod 3120)

d= 2753

Encrypt and decrypt using d and e.

For example plain text p= 65 can be encrypted as:

$C=65^{17}$ (mod 3233)= 2790

Cipher text C= 2790 can be decrypted as:

$P=2790^{2753}$ (mod 3233)= 65

This RSA cipher is quite complex, right? And now we learned one of the essential math that is used in Cryptography!

8. Conclusion

As you see, this cryptography is based on the mathematics. If mathematics wasn't used, such development of cryptography might not be possible. Yes, our whole world is filled with mathematics even though it is not appeared directly.

		hits	at bats	average
player 1	first half	41	200	**0.205**
	second half	31	120	**0.258**
	overall	72	320	0.225

		hits	at bats	average
player 2	first half	20	100	0.200
	second half	62	250	0.248
	overall	82	350	**0.234**

<Cited : picture from Wolfram Demonstration Project>

To best understand this paradox, I will start with an example.

Let's say there are two baseball players. Player 1 has a batting average 0.205 per 200 bats for the first half, and 0.258 per 120 bats for the second half. Player 2 has a batting average 0.200 per 100 bats for the first half, and 0.234 per 250 bats for the second half. From this, we can discern that player 1 has a higher batting average for both the first and second half (0.205 > 0.200 , 0.258 > 0.248) but has a lower overall batting rate (0.225 < 0.234). Simpson's paradox, therefore, represents a trend that appears in different groups of data but disappears or reverses when these groups are combined.

Let's modify this numbers and arrange into a formula. Let player 1, 2's batting rate in first half as $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$, let second half for $\frac{a_3}{a_4}$ and $\frac{b_3}{b_4}$. (meaning $\frac{hit}{atbat}$)

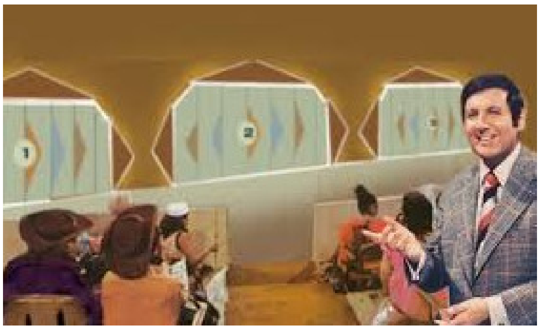
In the first half $\frac{a_1}{a_2} > \frac{b_1}{b_2}$, and in the second half $\frac{a_3}{a_4} < \frac{b_3}{b_4}$, so that we can easily conclude that $\frac{b_1+b_3}{b_2+b_4} > \frac{a_1+a_3}{a_2+a_4}$ is not always true.

Team	Overall	Indoor	Outdoor
Chicago Cubs	0.257	0.290	0.253
Oakland Athletics	0.256	0.292	0.255
Cleveland Indians	0.248	0.226	0.249
Tampa Bay Rays	0.247	0.243	0.251
Colorado Rockies	0.263	0.243	0.264
Milwaukee Brewers	0.262	0.252	0.265
Los Angeles Angels	0.248	0.236	0.249
Tampa Bay Rays	0.247	0.243	0.251
New York Mets	0.249	0.195	0.252
Toronto Blue Jays	0.248	0.225	0.255

<Real cases in America baseball league>

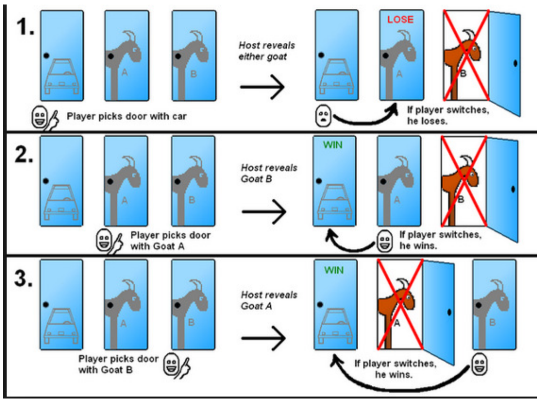
B. Monte hall dilemma

In America's famous TV show, "Monte Hall," there is a segment called "Let's Make a Deal." In this segment, a cast member may choose one door amongst three, and depending on this choice, can receive a prize. The situation goes like this:



There are three doors numbered 1 to 3. Behind one door there is a great car, whereas behind the other two, there is sheep. If the cast member chooses the door with the car, they get to keep it. However, if they choose a door with the sheep, they do not get anything.

Here is where the Monte Hall dilemma can be seen. Suppose the cast member chooses door 1. Before opening this door, the MC shows the cast member what is behind one of the other two doors. As the MC knows which door contains a sheep, the MC will open a door with the sheep behind it. The MC then asks the cast member whether they would like to change their choice. What would be the best choice for the cast member?



So, this monte hall dilemma solves easily when you think of all three possibilities. If the cast didn't change his choice, the probability that he will get a car is 1/2. But, there is probability of 1/3 at the first time. Then, it increases into 2/3 after MC open the door.

If you see the picture, you can fimly understand it. The three possibilities are shown in the picture. Therefore, if the cast changed his opinion, he will win the car for 2/3 of the probability.

By further research, I can find out the formual that were used in this concept.

If incident A happened, the conditional probability for incident B is P(B|A), the following fomula is always correct by using product rule.

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)P(A|B)}{P(A)}$$

(Let P(A), P(B), P(C) the probability that car will be in door A,B,C and P(OA), P(OB), P(OC) the probability that MC will open the door A,B,C)

So that can be apply into this example,

$$P(C|OB) = \frac{P(OB|C)P(C)}{P(OB)} = \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

$$P(B|OC) = \frac{P(OC|B)P(B)}{P(OC)} = \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

But the probability for door A (car is in door A) is 1/3.

$$P(A|OB) = \frac{P(OB|A)P(A)}{P(OB)} = \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(A|OC) = \frac{P(OC|A)P(A)}{P(OC)} = \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

Therefore, changing his choice would increase his probability for getting a car.

After this show, this was held in another TV show 'Ask Marilyn'. She said changing will increase the probability, and she got a lot of complains that she reduce the interest of the Monte Hall show. As we can see, the Monte hall dilemma is very famous problem in America.

3. Conclusion

By informing readers with these examples, I hope I have given them a bit of help in making better decisions when facing similar situations. By staying informed, we may improve our chances of making good decisions.

4. Citation

-http://blog.drscottfranklin.net/tag/simpsons-paradox/
-http://www.math.cornell.edu/~mec/2008-2009/TianyiZheng/Conditional.html

3. Geometric Series

A geometric sequence is one where the terms are in a ratio. This means that each next term is the previous multiplied by a ratio. Its recursive formula is $a_n = r * a_{n-1}$, where r is the constant ratio between the terms. For example, one such sequence is $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \frac{32}{243}, \ldots$ where $r = \frac{2}{3}$ and $a_1 = 1$. A series is the summation of a sequence, meaning that we want the sum of all the terms in a sequence. The series can also be infinite, meaning it never ends, such as $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \cdots$ forever. If $|r| < 1$, this series converges, so we can actually find the value of this summation. More generally, this is $a + ar + ar^2 + ar^3 + ar^4 + \cdots$ where a is the first term and r is the ratio between the terms. A nice formula for this converging infinite geometric series is $\frac{a}{1-r}$. For example, we know that $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \cdots = \frac{1}{1-\frac{2}{3}} = 3$ (since $a = 1$, and $r = \frac{2}{3}$).

4. Generating Functions and Their Uses

Now we will learn about the actual topic, generating functions. Formally stated, to describe a sequence $S = a_0, a_1, a_2, a_3, a_4 \ldots$, it is often useful to look at the series $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots = \sum_{n=0}^{\infty} a_n x^n$. They are called generating functions because we can generate surprising knowledge from the coefficients by skillfully manipulating these polynomials.

To make this more concrete, let us look at some examples of how generating functions can be used. Suppose that I have 13 pennies (1 cent), 7 nickels (5 cents), 4 dimes (10 cents), and 2 quarters (25 cents). How many different ways can I make 67 cents? This problem seems very difficult, but using generating functions, we have actually have an easy way to solve this problem!

Let the polynomial $1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13}$ represent the pennies. The exponents represent the various values I can create with just the pennies. I could use no pennies, one penny, two pennies, three pennies, all the way until I use all thirteen pennies. By the same reasoning, the polynomial $1 + x^5 + x^{10} + x^{15} + x^{20} + x^{25} + x^{30} + x^{35}$ represents the values I can create with just nickels (I can have 0, 1, 2, …, 7 nickels). Likewise, $1 + x^{10} + x^{20} + x^{30} + x^{40}$ represents the values I can create with the dimes and $1 + x^{25} + x^{50}$ represents the values I can create with the quarters. When we multiply the expressions together, we have

$$(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13})(1 + x^5 + x^{10} + x^{15} + x^{20} + x^{25} + x^{30} + x^{35})(1 + x^{10} + x^{20} + x^{30} + x^{40})(1 + x^{25} + x^{50}) =$$

$$1 + x + x^2 + x^3 + x^4 + 2x^5 + 2x^6 + \cdots + 24x^{66} + 24x^{67} + 24x^{68} + \ldots + x^{137} + x^{138}$$

Now just as before, the exponent represents the value, or the number of cents. The coefficient of every x-term represents how many ways I can achieve that value. Thus, we see from the $24x^{67}$ term, we can create 67 cents 24 ways.

Let's see a similar example. Suppose that I am at an amusement park and there are four colors of tickets available for rides. There are red tickets for \$1 each, blue tickets for \$2 dollars each, green tickets for \$3 each, and yellow tickets for \$4 each. Suppose I want an even number of red tickets, at least two blue tickets, an odd number of green tickets, and perhaps yellow tickets. I intend to use all of \$25 dollars. How many ways can I do this?

At first, the problem also seems extremely difficult. However, we can use generating functions to help us! I want an even number of red tickets and they cost \$1 each, so the cost of the red tickets can be either 0, 2, 4, 6, … Thus, we can represent the red tickets with $1 + x^2 + x^4 + x^6 + \cdots$. The blue tickets cost \$2 dollars each, and I want at least two. So the cost of the blue tickets can be 4, 6, 8, 10, …, and we have $x^4 + x^6 + x^8 + x^{10} + \cdots$. The green tickets cost \$3 each, and I want an odd number. The possible costs of just the green tickets can therefore be 3, 9, 15, 21, …, and we have $x^3 + x^9 + x^{15} + x^{21} + \cdots$. Finally, the yellow tickets have no restriction and cost \$4 each, so the possible costs are 0, 4, 8, 12, 16, … , and we have $1 + x^4 + x^8 + x^{12} + x^{16} + \cdots$.

When we multiply these together, we will obviously gain an infinite sum since all the polynomials continue forever. However, we only care about the coefficient of the x^{20} term. When we multiply all the polynomials together, we will eventually get $x^7 + 2x^9 + 4x^{11} + 7x^{13} + 11x^{15} + 16x^{17} + 23x^{19} + 31x^{21} + 41x^{23} + 52x^{25} + \cdots$. The $52x^{25}$ term tells us that there are 52 ways that I can use $25 dollars with the restrictions I placed.

5. Finding an Exact Formula for the Fibonacci Sequence

Believe or not, we now have all the tools to find an exact formula for the Fibonacci Sequence! We will use some advanced algebra, but we now understand the main tools we will use: generating functions, infinite series, and recursion. Let's get started. The Fibonacci Sequence defined recursively is $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. We care about the function $f(x) = \sum_{n=0}^{\infty} F_n x^n = 0 + x + x^2 + 2x^3 + 3x^4 + 5x^5 + 8x^6 + 13x\text{^}7 + \cdots$. The exponent tells us what term we are on, and the coefficient tells us the value of that term.

We can bring out two terms from our summation to get $f(x) = \sum_{n=0}^{\infty} F_n x^n = 0 + x + \sum_{n=2}^{\infty} F_n x^n$. You will see why this is useful shortly. Using our recursive formula, we get $f(x) = x + \sum_{n=2}^{\infty} F_n x^n = x + \sum_{n=2}^{\infty}(F_{n-1} + F_{n-2})x^n = x + \sum_{n=2}^{\infty} F_{n-1} x^n + \sum_{n=2}^{\infty} F_{n-2} x^n$. Now something very interesting will happen by bringing out some x-terms from our summation: $f(x) = x + \sum_{n=2}^{\infty} F_{n-1} x^n + \sum_{n=2}^{\infty} F_{n-2} x^n = x + x\sum_{n=2}^{\infty} F_{n-1} x^{n-1} + x^2 \sum_{n=2}^{\infty} F_{n-2} x^{n-2}$. Why? If you think about it, we can compute $\sum_{n=2}^{\infty} F_{n-1} x^{n-1} = x + x^2 + 2x^3 + 3x^4 + \cdots = f(x)$ and $\sum_{n=2}^{\infty} F_{n-2} x^{n-2} = 0 + x + x^2 + 2x^3 + 3x^4 + \cdots = f(x)$. Therefore, we have $f(x) = x + x\,f(x) + x^2 f(x)$. Solving for $f(x)$, we have $f(x) = \frac{x}{1-x-x^2}$. This is our generating function for the Fibonacci Sequence.

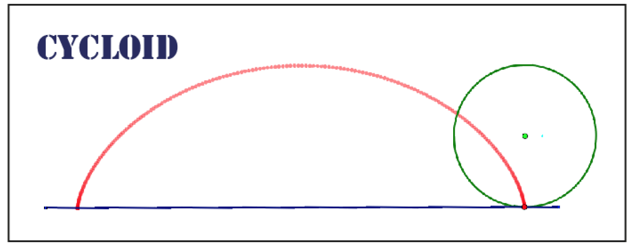
Now we want to be able to write $f(x)$ as the sum of two geometric series without recursion. To do this, we will write $f(x)$ using partial fractions. $1 - x - x^2 = (1 - \frac{1-\sqrt{5}}{2}x)(1 - \frac{1+\sqrt{5}}{2}x)$, so $f(x) = \frac{x}{(1-\frac{1-\sqrt{5}}{2}x)(1-\frac{1+\sqrt{5}}{2}x)} = \frac{A}{1-\frac{1-\sqrt{5}}{2}x} + \frac{B}{1-\frac{1+\sqrt{5}}{2}x}$ where A and B are some constants. Solving for A and B, we get that $f(x) = \frac{1}{\sqrt{5}}\left(\frac{1}{1-\frac{1-\sqrt{5}}{2}x} - \frac{1}{1-\frac{1+\sqrt{5}}{2}x}\right)$. The process for finding partial fractions is covered in any introductory calculus course if you want more information.

Recall that $a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1-r}$. Using this formula with $r = \frac{1-\sqrt{5}}{2}x$, we see that $\frac{1}{1-\frac{1-\sqrt{5}}{2}x} = 1 + \left(\frac{1-\sqrt{5}}{2}\right)x + \left(\frac{1-\sqrt{5}}{2}\right)^2 x^2 + \left(\frac{1-\sqrt{5}}{2}\right)^3 x^3 + \cdots = \sum_{n=0}^{\infty}\left(\frac{1-\sqrt{5}}{2}\right)^n x^n$ and similarly, $\frac{1}{1-\frac{1+\sqrt{5}}{2}x} = 1 + \left(\frac{1+\sqrt{5}}{2}\right)x + \left(\frac{1+\sqrt{5}}{2}\right)^2 x^2 + \left(\frac{1+\sqrt{5}}{2}\right)^3 x^3 + \cdots = \sum_{n=0}^{\infty}\left(\frac{1+\sqrt{5}}{2}\right)^n x^n$. Thus, $f(x) = \frac{1}{\sqrt{5}}\left(\frac{1}{1-\frac{1-\sqrt{5}}{2}x} - \frac{1}{1-\frac{1+\sqrt{5}}{2}x}\right) = \frac{1}{\sqrt{5}}\left(\sum_{n=0}^{\infty}\left(\frac{1-\sqrt{5}}{2}\right)^n x^n - \sum_{n=0}^{\infty}\left(\frac{1+\sqrt{5}}{2}\right)^n x^n\right) = \sum_{n=0}^{\infty}\frac{1}{\sqrt{5}}\left(\left(\frac{1-\sqrt{5}}{2}\right)^n - \left(\frac{1+\sqrt{5}}{2}\right)^n\right)x^n$. Finally, remember from the beginning of this section that we defined that $f(x) = \sum_{n=0}^{\infty} F_n x^n$. Then that means we have found $F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1-\sqrt{5}}{2}\right)^n - \left(\frac{1+\sqrt{5}}{2}\right)^n\right)$. F_n is the nth term of the Fibonacci sequence, so we have just found the exact formula!

6. Conclusion

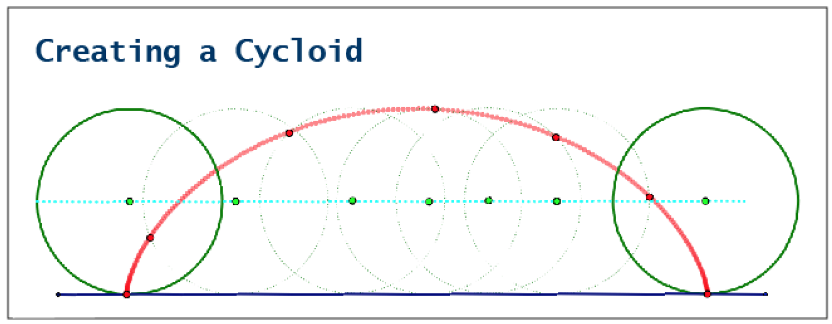
You should feel very proud having just derived for yourself a very complicated exact formula for the Fibonacci Sequence! Not only that, you now have a solid understanding of generating functions and an appreciation for how powerful of a tool they can be. I believe that generating functions are a clear example of not just how useful mathematics can be, but also how truly beautiful and amazing it is. I hope that you now feel this way as well. Thank you!

As you can sense from the definition, drawing a cycloid is very simple.
All you need to do is follow what the definition tells you to do.



To create your own, take these two steps.

1. Mark the point where the circle and line meets.
2. Rotate the circle on the straight line. As you do so, mark the path that the point takes.
    a. You can mark the path by fixing a pencil on the point so that it naturally creates a path as you rotate the circle.
    b. You can also mark the points at different intervals while rotating the circle and connect the points at the end to make the curve.



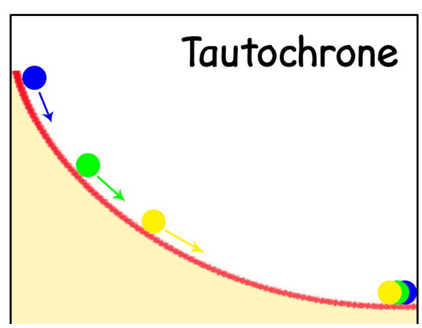
But this seemingly simple concept is not simple at all, earning the title, 'Helen of Geometry'.

Helen was a princess that caused the Troy War. Likewise, the cycloid causes frequent quarrels with even the founder of the cycloid still remaining a mystery.
With its many mysterious aspects, the curve's traits are unique and numerous as well.

3. Characteristics of a Cycloid

For now, let us focus on two main characteristics, that we will see being applied in our everyday lives.
These characteristics should be taken with a picture of an inverted cycloid. You will find that an upright cycloid will need energy for something on one point to even start moving.

The first is the obvious. As I have mentioned before, the cycloid is a way of connecting two points so that they take the shortest amount of time to travel. Hence, the cycloid has been given the name '*brachistochrone*'.



The second, and more unique characteristic gives the cycloid another name, called '*tautochrone*'. What this is is that wherever one starts on the curve, it will always fall to the bottom of the curve at the same time.

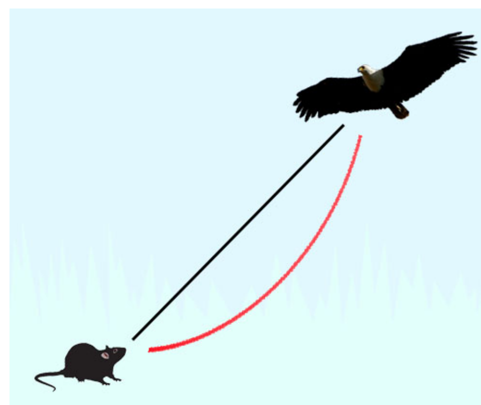
4. Cycloid in our Daily Lives

How are these cycloids seen in our daily lives? We will take a look into how cycloids have become part of nature through evolution. Cycloids can also be seen in the human society, from the time we did not even know about the concept of cycloids to after we found out about the characteristics and put it to use.

A. A Closer Look into Nature

There are a great number of animals and plants on our planet. Today, however, it is difficult to see many in our daily lives.

But one that we always see, even in the cities, is the bird. They waken us up with their songs and sing at regular intervals. As you may have already guessed what I am trying to get at, these birds use cycloids as well, although it is instinctive.



When birds see their prey, whether it is bugs for sparrows or mice for eagles, they have to get to the prey as quick as possible, or else the prey might run away or be taken by another predator. Hence, ***birds make a cycloid path from their position to the prey's position.*** These do not apply to all birds, however. It is usually seen in birds such as eagles and owls.

Cycloids can also be seen in the anatomy of birds and fish. They can also be seen in the ***scales of fish***. Especially, they are usually seen in bony fishes. These scales allow the fish to move faster through the water as the cycloids allow the water to pass quickly. For example trout, which lives in the fast waters have cycloid scales.

For us, these cycloids would take some amount of time to work out, as we would have to measure the distance, find the circle with the circumference matching the distance, and draw a cycloid. However, birds instinctively take cycloid paths instantly, without even having to think. Also, fishes are already born with these curves. This is able to take place through evolution, as animals try to maximize their ability to survive.

B. A Closer Look into Our World

In our technological world, we have made great use of cycloids. However, just like the instinctive natures of animals, our ancestors have used these concepts as well.

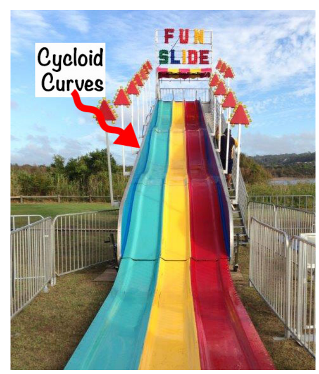
Let me first introduce you to ***Korean traditional roof tiles***.

Koreans made roof tiles that were curved in order to avoid decay. They had to find ways to make the roof not let in any water. They best way, they found would be to make the water run down the roof fast enough so that it would not seep in. While they could have made it straight, through many generations, they made the tiles curved, which resemble that of

the cycloid and minimized the level of decay.
A great example of the cycloid roof's effect showed, sadly, at a devastating incident at Sungnyemun. This place was built in the 1390s, as one of the eight gates in the fortress wall of Korea, and the oldest standing monument in Seoul.

On February 10th, 2008, a small fire started at Sungneymun. Even though the fire was found at an early stage, it could not be put out for five hours because of the cycloid tiles repelling water. Even though the water was poured, it simply 'rubbed off' too fast, and hence could not go in.
Our ancestors made great use of roof tiles, to keep their houses from decaying. The effect of cycloids demonstrated too well, however, and ended up in a disaster.
(On a side note, the Sungnyemun was rebuilt.)

Moving on from how our ancestors used cycloids, let us take a look at how we are using cycloids today, in our modern world.

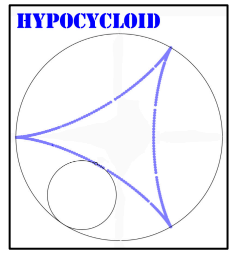


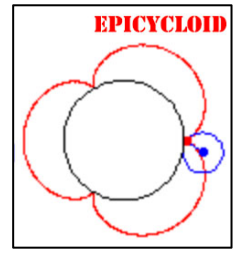
As I mentioned briefly in the beginning, cycloids are closer to us then we think it to be. A great example is a ***slide***.
There are many types of slides, but most have curves in them, which is more fun. This is because the curves are cycloids, which make us go down faster, letting us feel more thrill. The next time you go on a slide, see what kind it is. If there is a straight one and a curved one, try it out to feel which one is faster.

Another example can be seen in the ***path of spaceships***.
In order to get into space using the least amount of fuel and hence the quickest amount of time, spaceship's paths are usually in the form of a cycloid.

Lastly, these cycloids are used in ***gears in clocks.***
Gears, which are used in many machines, vary in type. Today, gears in most machines are involute gears since they are easier to make and cheaper. While they hold benefits, there are a lot of interruptions among these gears as the convex parts come in contact with the convex parts creating parts where they are not connected.
In the case of objects that require meticulousness such as clocks or cameras, there cannot be any interruptions.

Here, cycloid gears, which are harder to make are useful. To be exact, it is the cycloids called *epicycloids* (cycloids drawn not on a line but around the 'outside' of a circle), and *hypocycloids* (cycloids drawn around the 'inside' of the circle) that allows for no interruptions. These cycloids make up the flank (side of the teeth), with the convex epicycloids on top and concave hypocycloids on the bottom. Hence, unlike convex meeting convex, the epicycloid meets the hypocycloid, and fits perfectly continuously, which allows for no interruptions.





5. Conclusion

Cycloids are just one small part of the large world of mathematics. Rather than just seen in a math textbook however, cycloids are in our every day lives, helping and entertaining us. Cycloids are not the only math principles that are applied in our lives. With just a little amount of interest, you will find countless applications of different principles in math.

Now, take a *closer* look around. What do you see?

2015 WMI Student Seminars

Short Chain Fatty Acid Disinfection Effects on *Ascaridia* Eggs

Dahee Chung(daheechung@csia.hs.kr)

August 2015

1. Abstract

The environmental application of short chain fatty acids could potentially play a significant role in organic agriculture because they are approved for use in organic systems. It has also been shown that these acids when they come into contact with the eggs of the swine ascarid, *Ascaris suum*, have deleterious effects when the acids are at a pH below the pK_a of the given acid. *Ascaridia galli*, a related ascaridid parasite of chickens, can have significant health effects on laying hens and cannot be controlled by anthelmintics in organic facilities. Thus, the application of fatty acids to kill the eggs of this worm by topical application to caging, if successful, would be a useful adjunct for the production of organic chicken eggs. Thus, in this work, the methods used to kill *A. suum* eggs were applied to those of *A. galli*. Combinations of 1.5 M butanoic, pentanoic, or hexanoic acids in 18 mM Tween 20 were used in seven experiments: 1.5M butanoic, 1.5M pentanoic, 1.5M hexanoic, 1.5M butanoic and pentanoic, 1.5M butanoic and hexanoic, 1.5M pentanoic and hexanoic, and 1.5M butanoic, pentanoic, and hexanoic. In all experiments, eggs were exposed to the acids at 37℃ with agitation. Two methods were used to determine egg viability after treatment: a propidium iodide permeable-dye method and a larval development method. The propidium iodide assay was performed using eggs the day they were treated with acid, and were scored as non-viable if the internal cell within the egg fluoresced indication g propidium iodide permeability. The larval development assay was evaluated for the percentage of eggs that contained viable larvae

7 days after the eggs were washed free of acid and placed in culture at 28C. As performed, the propidium iodide assay was not successful in identifying the majority of dead eggs when compared to the larval development assay. The larvation assay did show that the treatment of the eggs with the acids markedly reduce viability with 5 minutes of contact, and with 60 minutes contact the combinations acids all reduced egg viability by 97%. The work suggests that optimization of the application methods, contact time, temperature, and surfactant may allow this to become a viable means of *A. galli* control in organic facilities.

2. Introduction

The nematode *Ascaridia Galli* is a large worm, whose infection causes weight depression in the host (W. Malcolm Reid et al. 1958). One of the most striking effects of infection by *Ascaridia Galli* is the occasional finding of this parasite inside chicken's egg. Numerous reports of this phenomenon have been made in the literature (W. M., J. L. Mabon 1973). It is suggested that the worms migrate up the oviduct via the cloaca, with subsequent inclusion in the egg (Calnek, B. W 1997). Organic facilities can't use Anthelmintic to control these parasites. It is shown that these acids when they come into contact with the eggs of the swine ascarid, *Ascaris suum*, have deleterious effects when the acids are at a pH below the pK_a of the given acid(Butkus, M. A et al 2011). Thus, usage of short chain fatty acids in controlling the viability of *Ascaridia Galli* can be helpful in organic facilities since short chain fatty acids are approved for use in organic facilities. Two different hypotheses are constructed in this work: Short chain fatty acids will reduce the viablility of *Ascaridia* eggs and propidium iodide dye permeability will identify viable *Ascaridia* eggs without the need to wait seven days until eggs larvate.

3. Materials and Methods

Several tests involving various treatments were administrated using 950µl of the purposed treatment as well as 50µl of the *Ascaridia* egg sample which has concentration of 23.2 eggs/µl, purified from chicken feces. Thus, 1160 eggs are used in each samples.

Combinations of 1.5 M butanoic, pentanoic, or hexanoic acids in 18 mM Tween 20 were used in seven experiments: 1.5M butanoic, 1.5M pentanoic, 1.5M hexanoic, 1.5M butanoic and pentanoic, 1.5M butanoic and hexanoic, 1.5M pentanoic and hexanoic, and 1.5M butanoic, pentanoic, and hexanoic.

After eggs were added to acid solution for a final volume of 1 ml in 1.2ml microfuge tube, they were vortexed for three seconds and placed in a heating block at 37℃ with agitation. At various sampling times, the tubes were removed and centrifuged at centrifugal force1200g for a minute to pellet the eggs. The acid was suctioned off without disturbing the egg pellet, and the eggs were washed six times with phosphate buffer (pH 7.0, 10mM). Unless otherwise stated all experiments were carried out in two replications for each exposure time, temperature, acid concentration and acid with surfactant.

Method 1 Propidium Iodide Permeable Dye Assay

Half of the eggs in microfuge tube were stained with Propidium Iodine. Propidium Iodide is fluorescent dye, binding intercalatively to both DNA and RNA with little base-pair specificity. Pink Propidium Iodide Fluorescence under UV excitation is indicative of cell permeabilization. Both DNA and RNA are stained, giving a red-pink luminescence under excitation form a triple-filter set. Because this dye permeates outer shell and nuclear membrane to get through DNA, we can notice the viability of *Ascaridia* eggs by looking inside fluorescence microscope. Figure 1 shows the difference in color of viable *Ascaridia* eggs and non-viable *Ascaridia* eggs in fluorescence.

Method 2 Larvation

After washing, a half of all eggs were transferred to 12 well culture plates with the addition of H_2SO_4 solution to retard mold growth during incubation. The plate, wrapped in a wet paper towel in a plastic box, was statically incubated at 28℃ (82.4℉) for 7 days. The eggs were counted and scored as larvated (viable) or nonlarvated (not viable). Figure 2 demonstrates the daily egg development of *Ascaridia* egg from day 0 to day 19 showing the larvated form of *Ascaridia* egg. Figure 3 shows the difference in existence of larvae in viable *Ascaridia* eggs and non-viable *Ascaridia* eggs.

4. Results

Method 1 Propidium Iodide Permeable Dye Assay

As shown in Figure 4, the viability of *Ascaridia* egg decreases as exposure time pasts by. Compared to control, butanoic, pentanoic, and hexanoic acids all reduced the viability of the *Ascaridia* eggs to below detectable limits. However, there is certainly a viability difference between pentanoic acid and even-number acids, which are butanoic acid and hexanoic acid. While viability of *Ascaridia* eggs in pentanoic acid exposures at 60 minutes is about 88%, the viability of *Ascaridia* eggs in butanoic acid exposures is 81% and the viability of *Ascaridia* eggs in hexanoic acid exposure is 82%.

Continuing to the results of each acid exposure, Figure 5 which is the result of combination of each two acids exposure also shows the decrease in viability of *Ascaridia* eggs compared to control. Also shown in Figure 4, pentanoic acid has less effect on decreasing viability on *Ascaridia* eggs. The combination of different types of fatty acids actually decreases the viability of *Ascaridia* Eggs that viability of *Ascaridia* eggs in butanoic and pentanoic acid exposures at 60 minutes is 82% and viability of *Ascaridia* eggs in pentanoic and hexanoic acid exposures at 60 minutes is 79%. Without pentanoic acid, viability of *Ascaridia* eggs in butanoic and hexanoic acid exposures at 60 minutes is the least, 74%.

As shown in Figure 6, Combination of all three acids gave us similar result that viability of *Ascaridia* eggs in butanoic, pentanoic and hexanoic acid exposures at 60 minutes is 75% while the viability of *Ascaridia* eggs in control at 60 minutes is about 90%. Although we could gain data right after the treatments are done to *Ascaridia* eggs, the viability data by propidium iodide permeable dye assay was not significant enough.

Method 2 Larvation

Different from that of viability by propidium iodide permeable dye assay, Figure 7,8,9, graphs of viability by larvation, have the axis dealing 0% to 100%. That is, the result shows more decreasing effects of fatty acids in viability of *Ascaridia* eggs. Figure 7 suggests that viability of *Ascaridia* eggs exposed on pentanoic acid is the highest for 24% at 60 minutes exposure. It means the lowest decrease on viability since the viability of *Ascaridia* eggs exposed on butanoic acid is 5.37% at 60 minute exposure and viability of *Ascaridia* eggs exposed on hexanoic acid is 5.62% at 60 minutes exposure.

As shown in Figure 8, results of combination of each two acids exposure also reveals high decrease in viability of *Ascaridia* eggs. In this result, the viability of *Ascaridia* eggs exposed in butanoic and hexanoic acid is the lowest which records 4.4% at 60 minute exposure. The viability of *Ascaridia* eggs in butanoic and pentanoic acid exposures is 6% and the viability of *Ascaridia* eggs in pentanoic and hexanoic acid exposure is 8%. All three results definitely show that the fatty acids actually have the effect on *Ascaridia* eggs' viability.

Figure 9 also clearly demonstrates the difference of *Ascaridia* egg viability between the control and butanoic, pentanoic, and hexanoic acid combination exposure. While 90% of eggs in the control survived after 60 minutes exposure, only 3% of eggs in butanoic, pentanoic and hexanoic exposure survived after 60 minute exposure at 37 °C in shaking condition.

5. Discussion

In conclusion, fatty acids exposure has decreased the viability of the *Ascaridia* eggs. The propidium iodide assay was not successful in identifying the majority of dead eggs when compared to the larval development assay. Even number acids, butanoic and hexanoic acid, are more effective than pentanoic acid in decreasing the viability of *Ascaridia* Eggs. These findings are consistent with those of Paggi and Fay (1996), where acetic acid was less effective than propanoic and butanoic acids against *Streptococcusbovis*. For the future studies, we should further test the fatty acid disinfection effects on *Ascaridia* eggs in manure matrix and contaminated egg producing equipment. According to the result, it is possible to develop an organic pesticide that can be sprayed in the cages of the chickens using fatty acids.

6. References

Reid, W. Malcolm, and James L. Carmon. "Effects of Numbers of *Ascaridia Galli* in Depressing Weight Gains in Chicks." *The Journal of Parasitology* 44.2 (1958): 183. Web.

Reid, W. M., J. L. Mabon, and W. C. Harshbarger. "Detection of Worm Parasites in Chicken Eggs by Candling." *Poultry Science* 52.6 (1973): 2316-324. Web.

Calnek, B. W. *Diseases of Poultry*. Ames, IA, USA: Iowa State UP, 1997. Print.

Butkus, M. A., K. T. Hughes, D. D. Bowman, J. L. Liotta, M. B. Jenkins, and M. P. Labare. "Inactivation of Ascaris Suum by Short-Chain Fatty Acids." *Applied and Environmental Microbiology* 77.1 (2010): 363-66. Web.

Paggi, R. A., and J. P. Fay. 1996. Effect of short-chain fatty acids on growth of the ruminal bacterium *Streptococcus bovis*. J. Gen. Appl. Microbiol. 42:393–400.

7. Figures Legends

Figure 1. Viable *Ascaridia* eggs (left) and non-viable *Ascaridia* eggs (right) stained in propidium iodide

Figure 2. Daily egg development of *Ascaridia* egg from day 0 to day 19

Figure 3. Larvated *Ascaridia* eggs (left) and Non-larvated *Ascaridia* eggs (right) in

Figure 4. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic, 1.5M pentanoic, 1.5M hexanoic, each in 18 mM Tween 20, by propidium iodide permeable dye assay

Figure 5. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic and pentanoic, 1.5M butanoic and hexanoic, 1.5M pentanoic and hexanoic, each in 18 mM Tween 20 by propidium iodide permeable dye assay

Figure 6. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic, pentanoic, and hexanoic in 18 mM Tween 20, by propidium iodide permeable dye assay

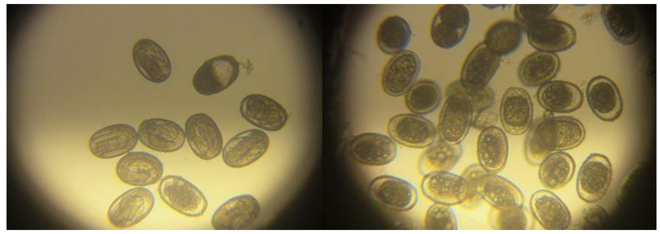
Figure 7. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic, 1.5M pentanoic, 1.5M hexanoic, each in 18 mM Tween 20, by larval development assay

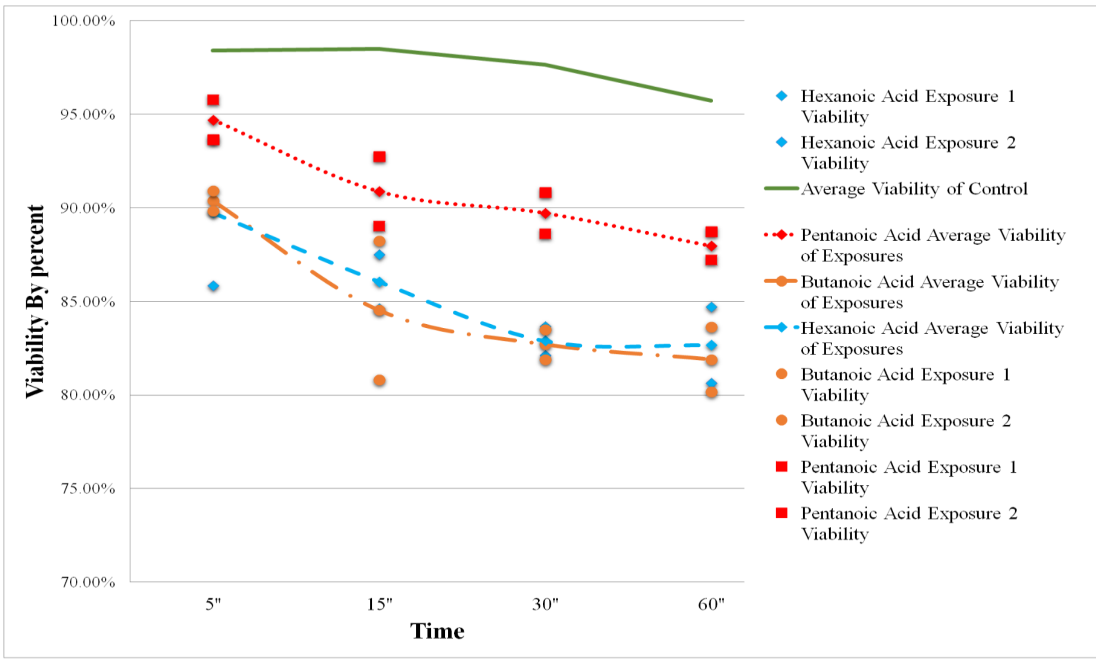
Figure 8. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic and pentanoic, 1.5M butanoic and hexanoic, 1.5M pentanoic and hexanoic, each in 18 mM Tween 20 by larval development assay

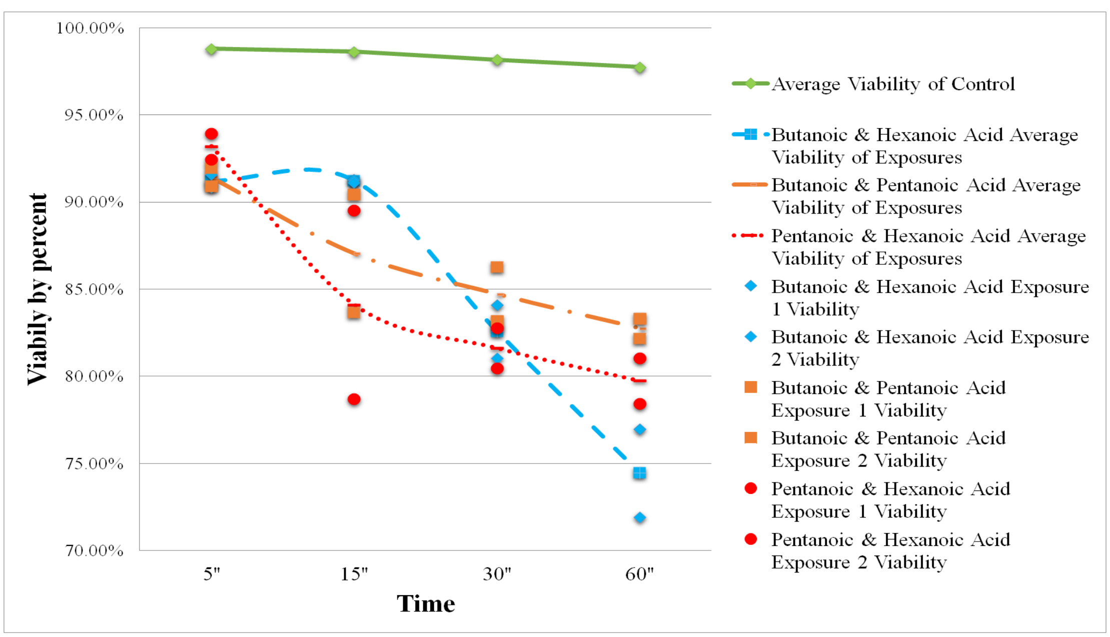
Figure 9. Graph of viability of *Ascaridia* egg exposed to 1.5M butanoic, pentanoic, and hexanoic in 18 mM Tween 20, by larval development assay

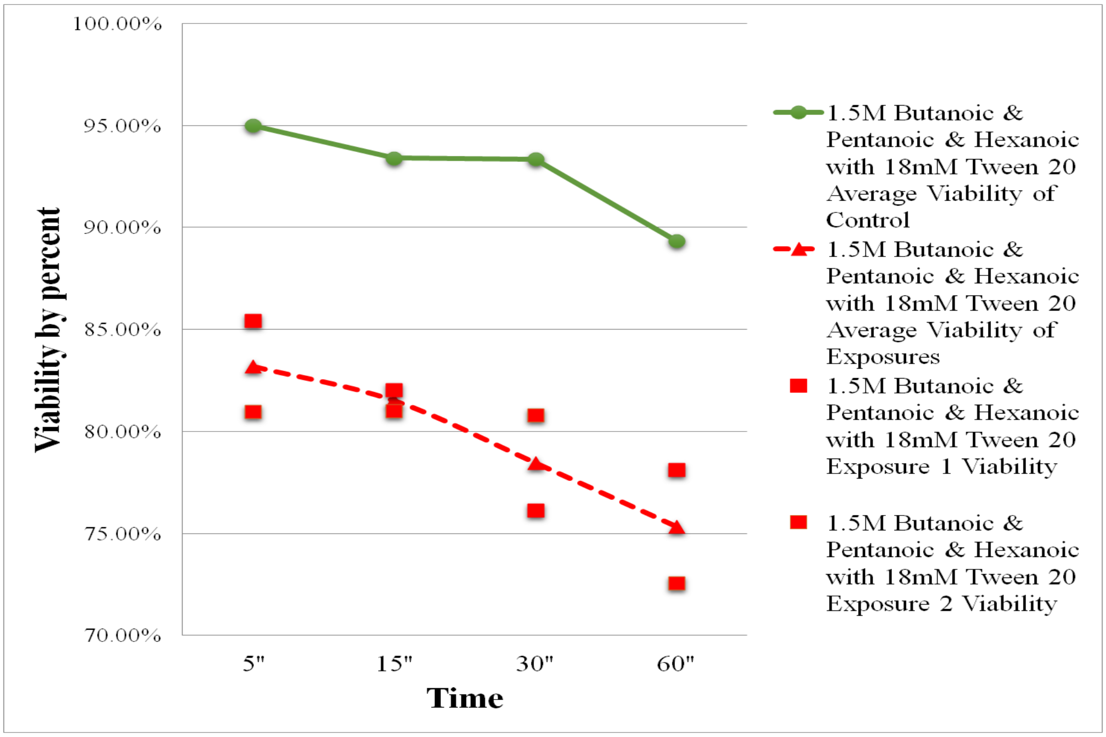
8. Figures

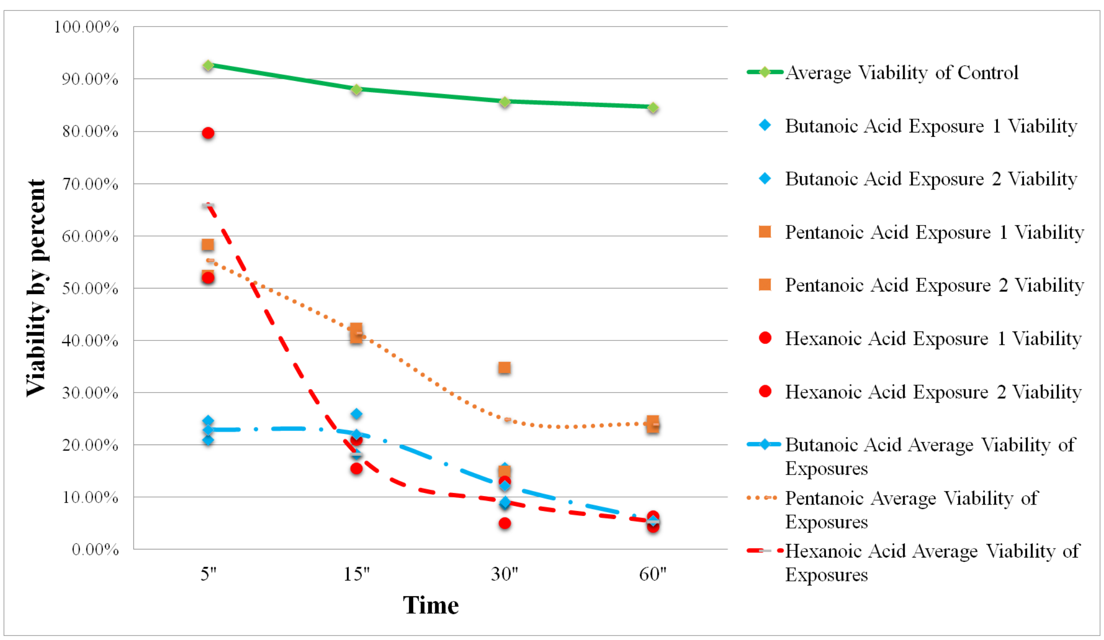

0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19

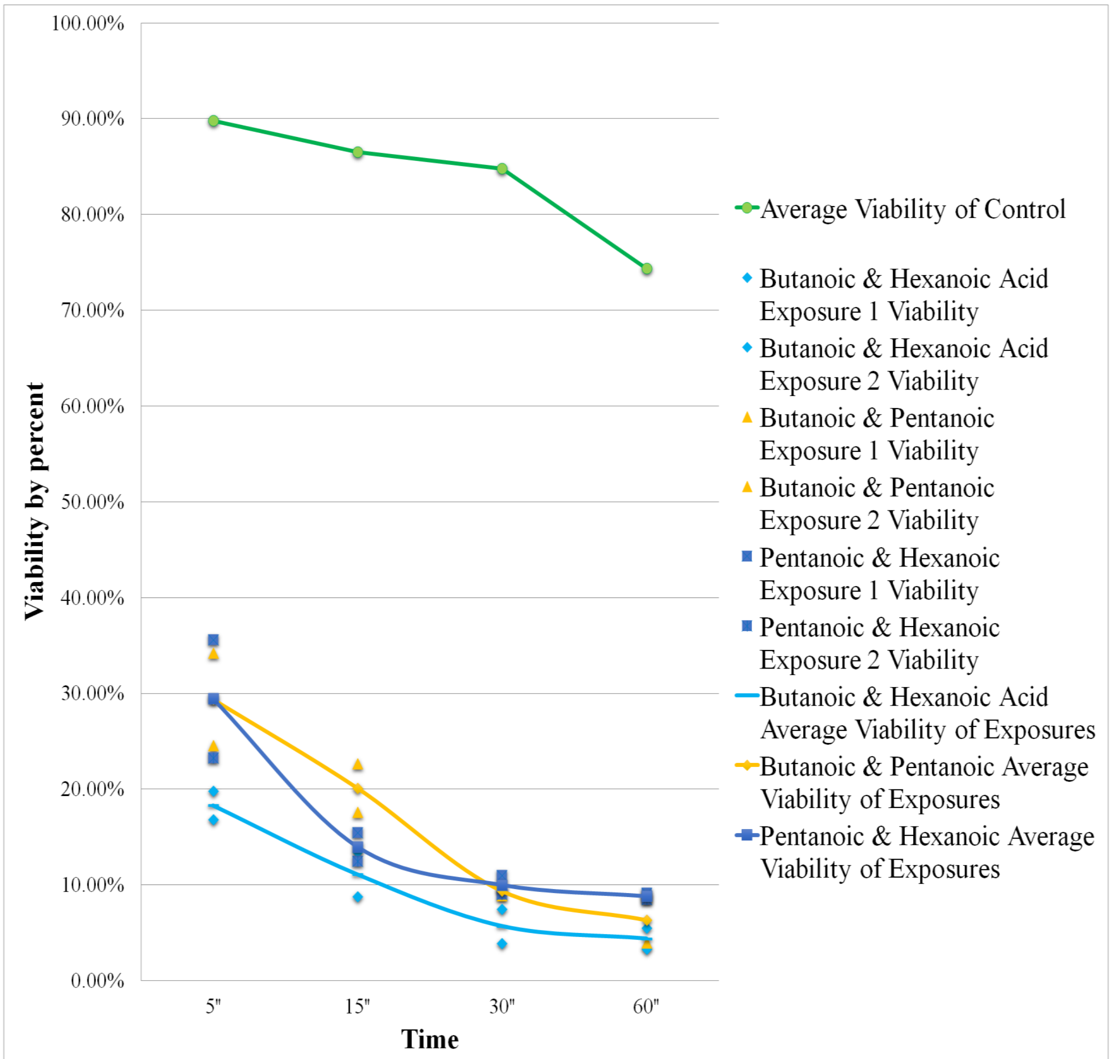



100.00%
95.00%
90.00%
85.00%
80.00%
75.00%
70.00%
Viability By percent
5"
15"
30"
60"
Time
Hexanoic Acid Exposure 1 Viability
Hexanoic Acid Exposure 2 Viability
Average Viability of Control
Pentanoic Acid Average Viability of Exposures
Butanoic Acid Average Viability of Exposures
Hexanoic Acid Average Viability of Exposures
Butanoic Acid Exposure 1 Viability
Butanoic Acid Exposure 2 Viability
Pentanoic Acid Exposure 1 Viability
Pentanoic Acid Exposure 2 Viability

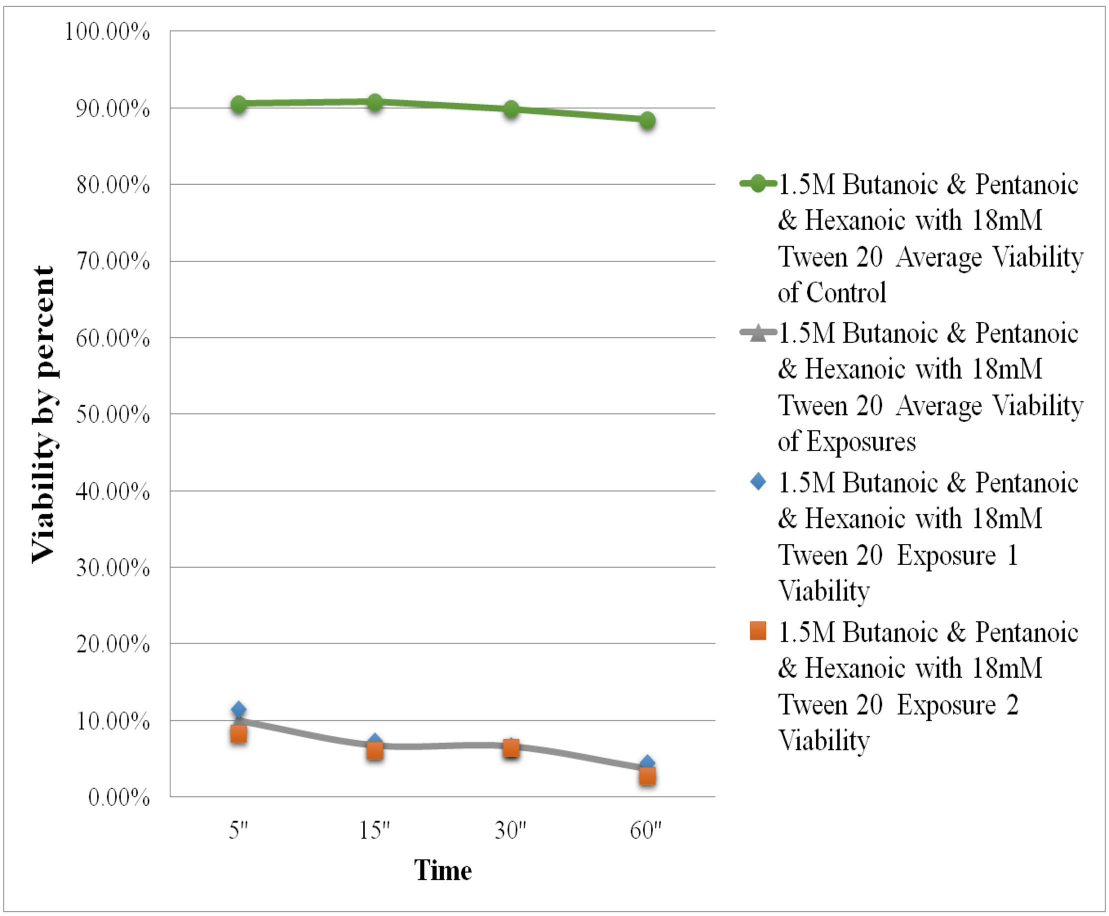

100.00%
95.00%
90.00%
85.00%
80.00%
75.00%
70.00%
Viabily by percent
5"
15"
30"
60"
Time
Average Viability of Control
Butanoic & Hexanoic Acid Average Viability of Exposures
Butanoic & Pentanoic Acid Average Viability of Exposures
Pentanoic & Hexanoic Acid Average Viability of Exposures
Butanoic & Hexanoic Acid Exposure 1 Viability
Butanoic & Hexanoic Acid Exposure 2 Viability
Butanoic & Pentanoic Acid Exposure 1 Viability
Butanoic & Pentanoic Acid Exposure 2 Viability
Pentanoic & Hexanoic Acid Exposure 1 Viability
Pentanoic & Hexanoic Acid Exposure 2 Viability


100.00%
95.00%
90.00%
85.00%
80.00%
75.00%
70.00%
Viability by percent
5"
15"
30"
60"
Time
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Average Viability of Control
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Average Viability of Exposures
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Exposure 1 Viability
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Exposure 2 Viability


100.00%
90.00%
80.00%
70.00%
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
Viability by percent
5"
15"
30"
60"
Time
Average Viability of Control
Butanoic Acid Exposure 1 Viability
Butanoic Acid Exposure 2 Viability
Pentanoic Acid Exposure 1 Viability
Pentanoic Acid Exposure 2 Viability
Hexanoic Acid Exposure 1 Viability
Hexanoic Acid Exposure 2 Viability
Butanoic Acid Average Viability of Exposures
Pentanoic Average Viability of Exposures
Hexanoic Acid Average Viability of Exposures



100.00%
90.00%
80.00%
70.00%
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
Viability by percent
5"
15"
30"
60"
Time
Average Viability of Control
Butanoic & Hexanoic Acid Exposure 1 Viability
Butanoic & Hexanoic Acid Exposure 2 Viability
Butanoic & Pentanoic Exposure 1 Viability
Butanoic & Pentanoic Exposure 2 Viability
Pentanoic & Hexanoic Exposure 1 Viability
Pentanoic & Hexanoic Exposure 2 Viability
Butanoic & Hexanoic Acid Average Viability of Exposures
Butanoic & Pentanoic Average Viability of Exposures
Pentanoic & Hexanoic Average Viability of Exposures


100.00%
90.00%
80.00%
70.00%
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
Viability by percent
5"
15"
30"
60"
Time
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Average Viability of Control
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Average Viability of Exposures
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Exposure 1 Viability
1.5M Butanoic & Pentanoic & Hexanoic with 18mM Tween 20 Exposure 2 Viability